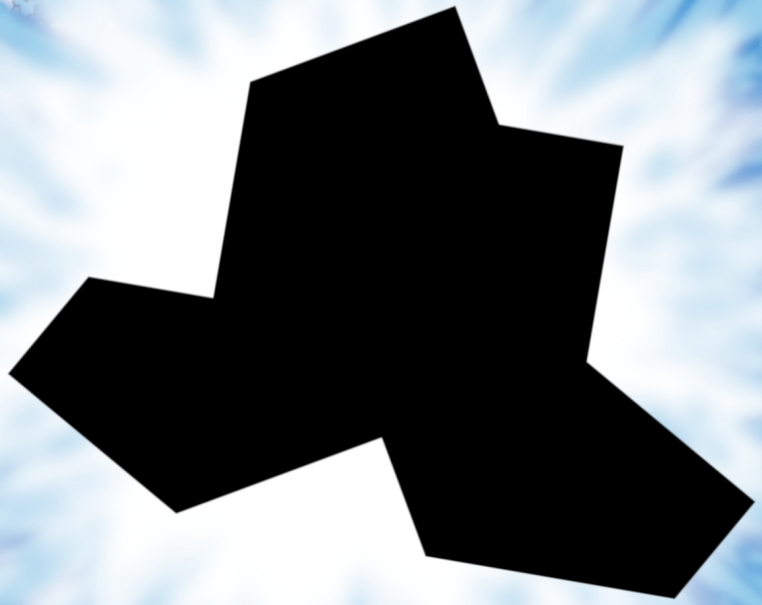
Con ustedes, un...



¿sombrero?

José Ezequiel Soto Sánchez

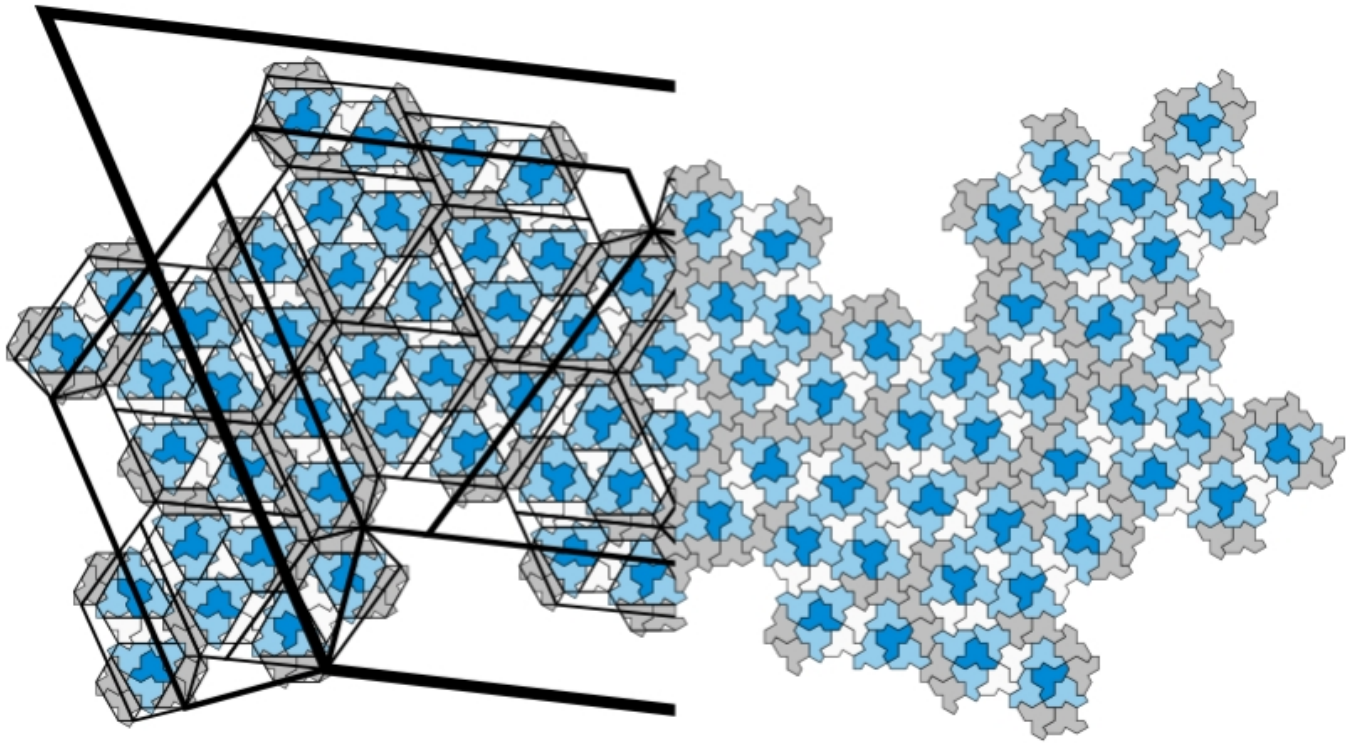
COLOQUIO DE MATEMÁTICAS

Mathematics > Combinatorics

[Submitted on 20 Mar 2023]

An aperiodic monotile

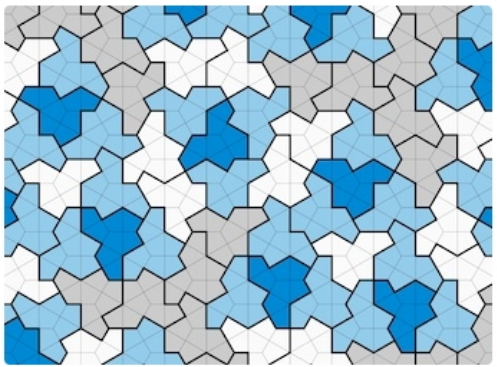
David Smith, Joseph Samuel Myers, Craig S. Kaplan, Chaim Goodman-Strauss



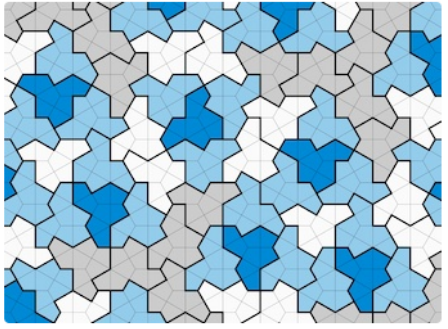
https://cs.uwaterloo.ca/~csk/hat/

A groundbreaking new discovery in mathematics!

The Hat, an aperiodic monotile



Presenting “the Hat,” a newly discovered (and first-ever!) shape that can tile the plane endlessly but only without ever quite repeating the pattern.



Presenting “the Hat,” a newly discovered (and first-ever!) shape that can tile the plane endlessly but only without ever quite repeating the pattern.

Hear the story behind the discovery

A Hat for Einstein

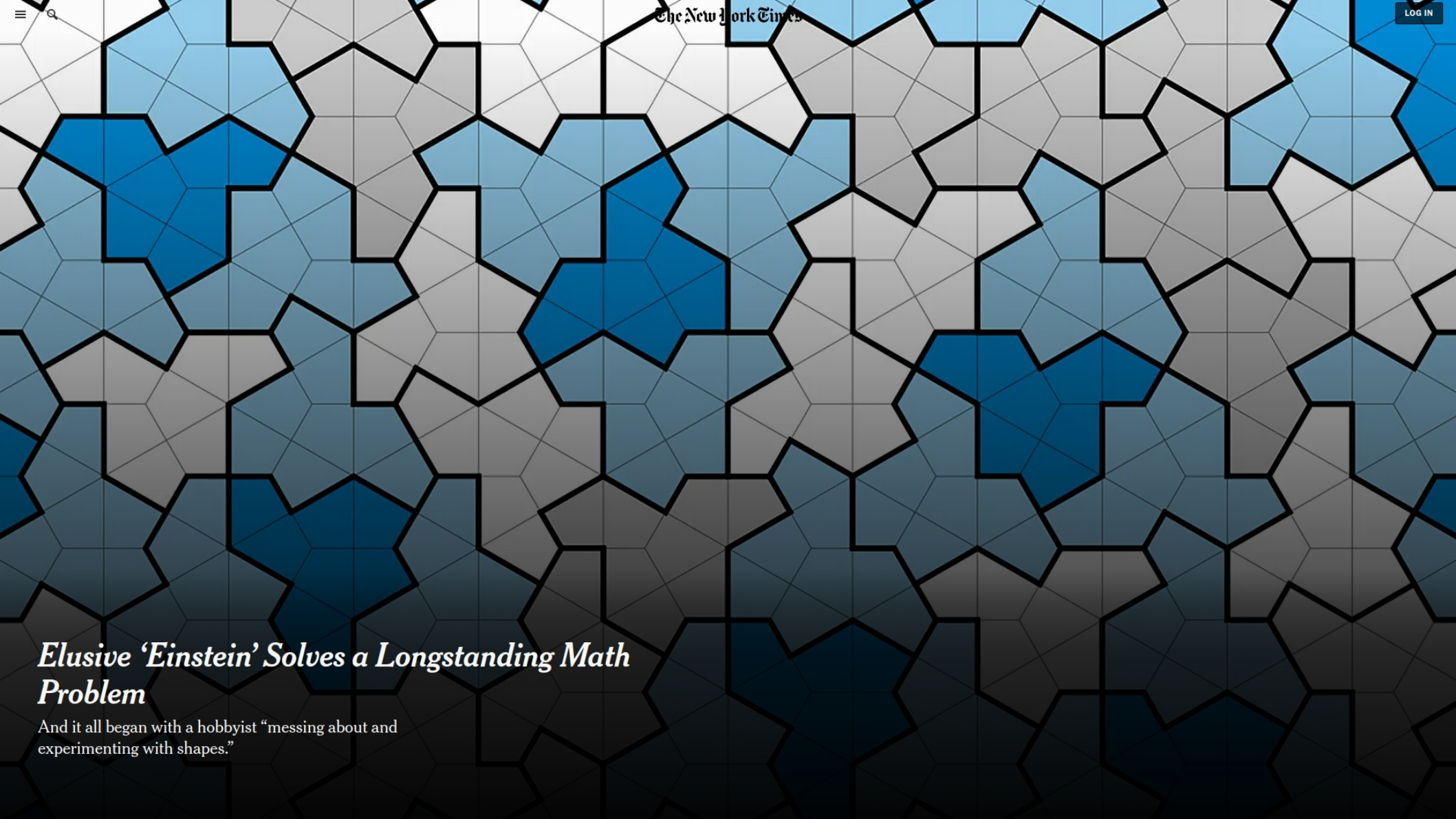
Sunday, March 25 at 3:00 pm ET

Join Craig S. Kaplan and Chaim Goodman-Strauss who, along with their co-authors David Smith and Joseph Samuel Myers, wrote the groundbreaking paper about the Hat. They will discuss how they discovered this aperiodic monotile and how it impacts modern mathematics. Watch the video.

Meet the Authors

Wednesday, March 29 at 6:00 pm ET

Join all four authors of the groundbreaking paper, David Smith, Joseph Samuel Myers, Craig Kaplan, and Chaim Goodman-Strauss, as they discuss their exciting mathematical discovery of the Hat, the first-ever shape that can tile the plane endlessly but only without ever quite repeating the pattern. Have your questions answered — or just enjoy the lively discussion — at this interactive Q&A session! Watch the video.



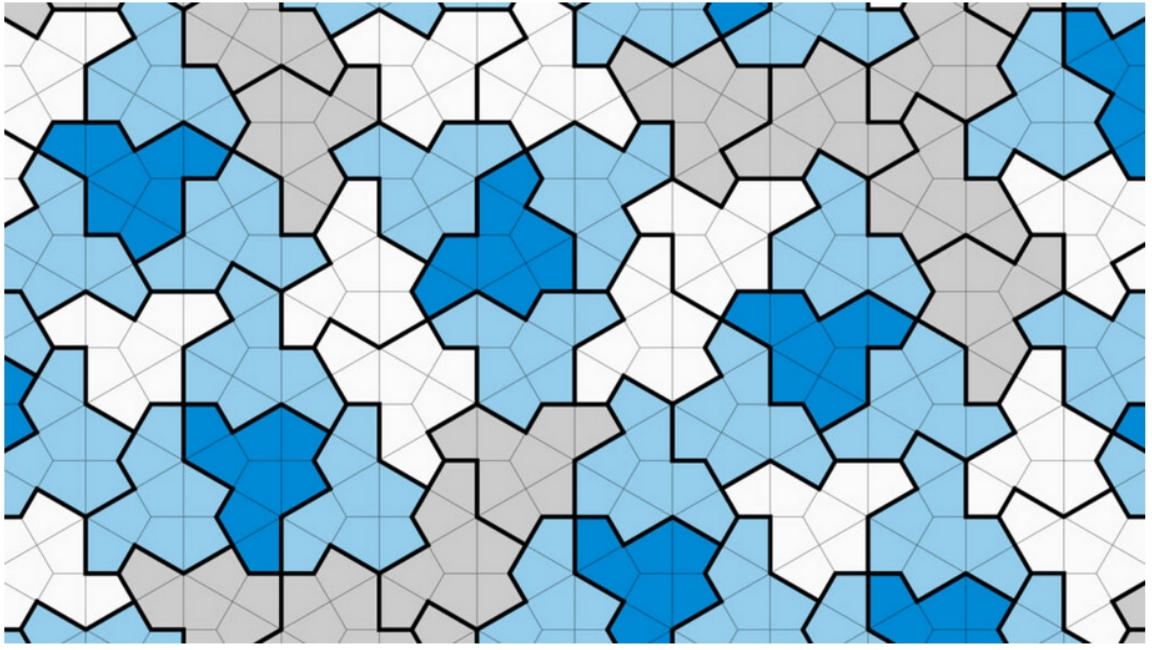
Elusive 'Einstein' Solves a Longstanding Math Problem

And it all began with a hobbyist "messing about and experimenting with shapes."

NEWS MATH

Mathematicians have finally discovered an elusive 'einstein' tile

A 13-sided shape called 'the hat' forms a pattern that never repeats



A 13-sided tile called "the hat" forms a pattern that covers an infinite plane yet it cannot repeat, making it a long-sought shape known as an "einstein." A sample of that pattern is shown here.

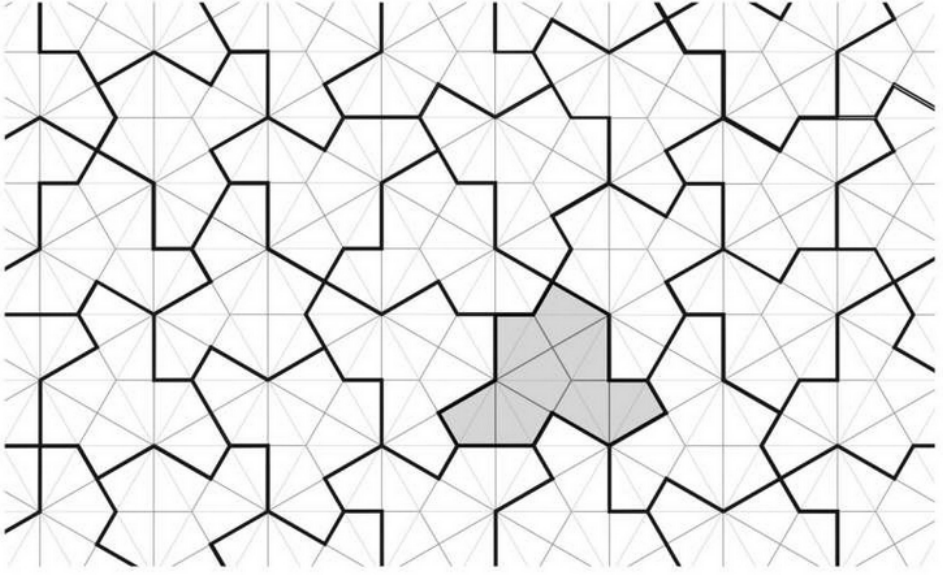
D. SMITH, J.S. MYERS, C.S. KAPLAN AND C. GOODMAN-STRAUSS (CC BY 4.0)

MARCH 23, 2023 REPORT

Editors' notes

A geometric shape that does not repeat itself when tiled

by Bob Yirka , Phys.org



The gray “hat” polykite tile is an “einstein”, an aperiodic monotile. In other words, copies o...

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GEOMETRY

Hobbyist Finds Math's Elusive 'Einstein' Tile

Erica Klarreich
Contributing Correspondent

April 4, 2023

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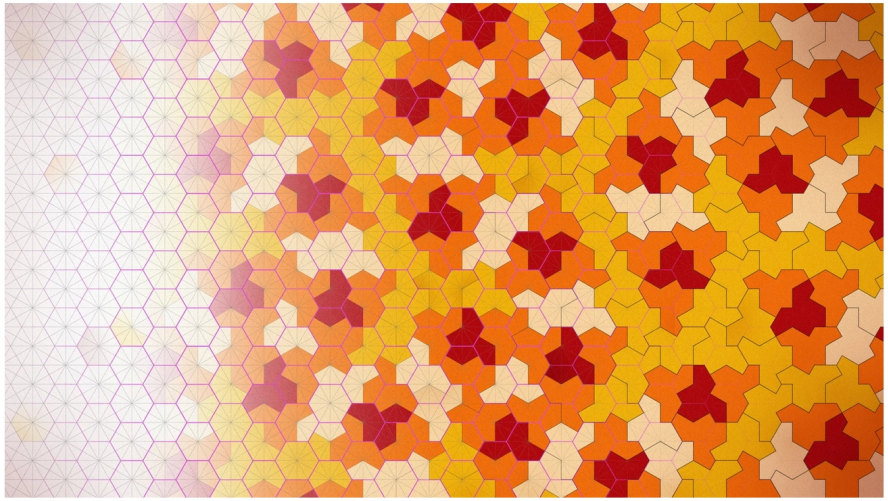
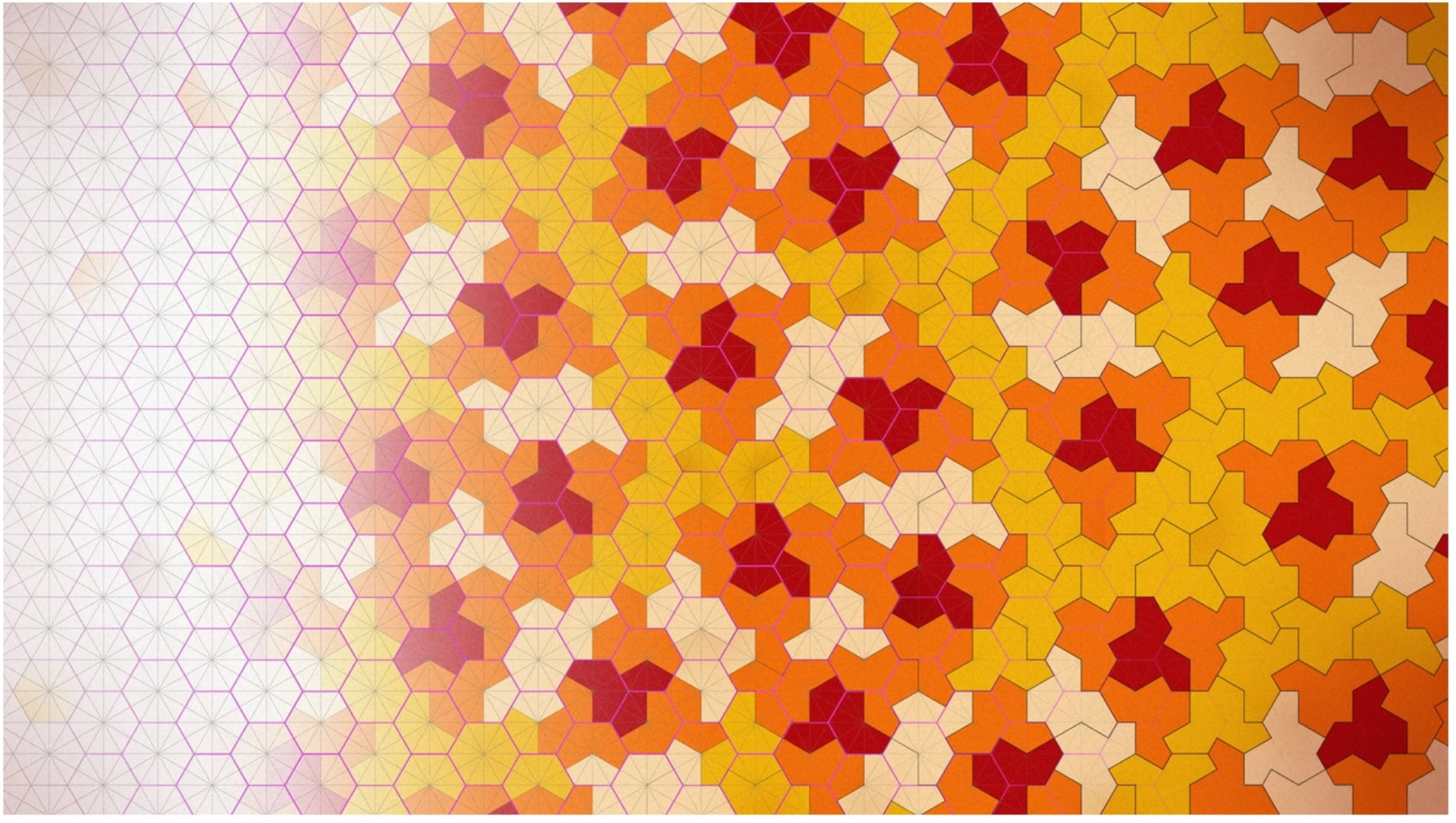
geometry mathematics symmetry All topics →

The surprisingly simple tile is the first single, connected tile that can fill the entire plane in a pattern that never repeats — and can't be made to fill it in a repeating way.



To make the tiling, slice an ordinary hexagonal grid into segments, then glue together adjoining segments.

Samuel Velasco/Quanta Magazine



To make the tiling, slice an ordinary hexagonal grid into segments, then glue together adjoining segments.

Samuel Velasco/Quanta Magazine

Hat Trick

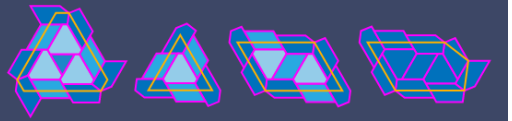
The proof that the hat tile is aperiodic relies on recursively creating ever-larger structures, a technique that goes back to the 1960s.

METATILES

Hat tile
Using only the hat tile, create four metatiles (named H,T,P,F).

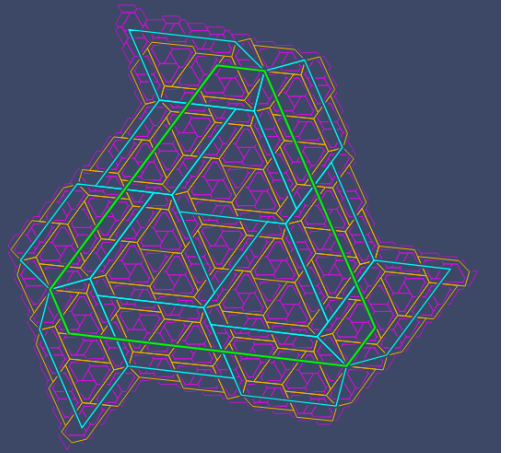
SUPERTILES

The metatiles can be further compiled into four larger versions of themselves called supertiles.



FOUR ITERATIONS

The four **supertiles** can be further arranged into even larger **supertiles** and so on, gradually filling the entire plane.



Craig Kaplan, a computer scientist at the University of Waterloo in Canada.

—

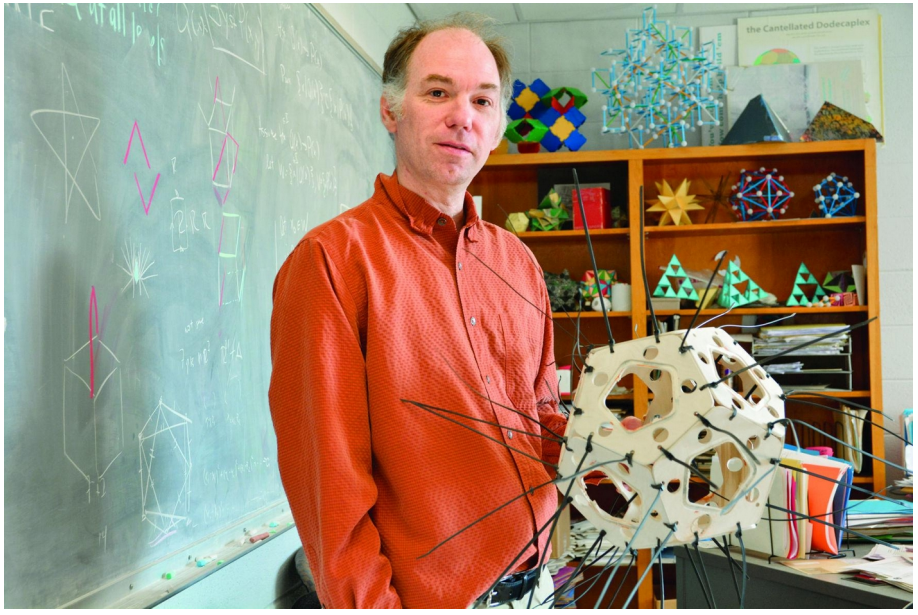
Joe Petrik

David Smith's discovery has been called "mind-boggling."

—

Courtesy of David Smith

https://www.quantamagazine.org/hobbyist-finds-maths-elusive-einstein-tile-20230404/



Chaim Goodman-Strauss

Joseph Myers

- Information on contacting me.
- Software I maintain or maintained: GCC, glibc, GNU toolchain and C standards development (active), matholymp (active), bsd-games (inactive since 2005, others may have newer versions), nppt (inactive since 2004).
- My publications and preprints.
- My mathematics.
- I have been involved in some manner in Mathematical Olympiad activities since 1992.
  - I am the current maintainer of the web site for the British Mathematical Olympiad Subtrust (BMOS) and Committee (BMOC), and am a member of BMOC and BMOS.
  - I maintain the web site for the European Girls' Mathematical Olympiad (and do associated EGMO IT work), and that for EGMO 2012, and was a member of the organising committee of EGMO 2012, which was held in Cambridge in April 2012.
  - I maintain the *UK IMO Register*, a listing of the details of past UK team members in the International Mathematical Olympiad (including both Cambridge's 1998 Fields Medalists).
- I am or have been involved in some Cambridge student societies, and for some of these have made relevant documents available online:
  - As Eureka Online Editor for the Archimedeans (Cambridge University Mathematical Society), I made the first parts of Eureka Online available.
  - As Archivist for the Archimedeans, I have made old versions of the Constitution, Standing Orders and Procedures available.
  - I am a member of the Trinity Mathematical Society, and updated and made available online Paul Taylor's list of past Trinity Mathematical Society meetings (older talks list also available).
  - I am a member of CUAS (Cambridge University Astronomical Society); I have never held any office in that society but have occasionally used the telescopes and have made available here information on the history of the Northumberland Telescope.
  - I've done a fair amount of work on the CUCPS (Cambridge University Computer Preservation Society) web site, in particular the information about TITAN. Note that CUCPS is no longer active.
- Historical information on British Summer Time.
- As part of my interest in computer history, I have also made available here the standard ECMA-10—ECMA standard for Data Interchange on Punched Tape. The standards organisation ECMA produce computer-related standards that are freely available both online and in printed form (even a complete set of current printed ECMA standards is free); unfortunately they cannot supply copies of obsolete standards such as this one, but since their standards are not copyright it can be distributed here. Many ECMA standards are essentially identical to expensive ISO ones; there are several ISO standards for punched paper tape (numbers 1113, 1154, 1729, 2195 and 3692) but I have not checked their equivalence to this one.

Contact: Joseph Myers (jsm@polyomino.org.uk)
Last updated: 25 November 2022

IT'S A SHAPE JIM, BUT NOT AS WE KNOW IT

Thanks to the super human effort of Craig Kaplan, Joseph Myers and Chaim Goodman-Strauss, the hat and turtle polykites have finally arrived on the scene… https://arxiv.org/abs/2303.10798

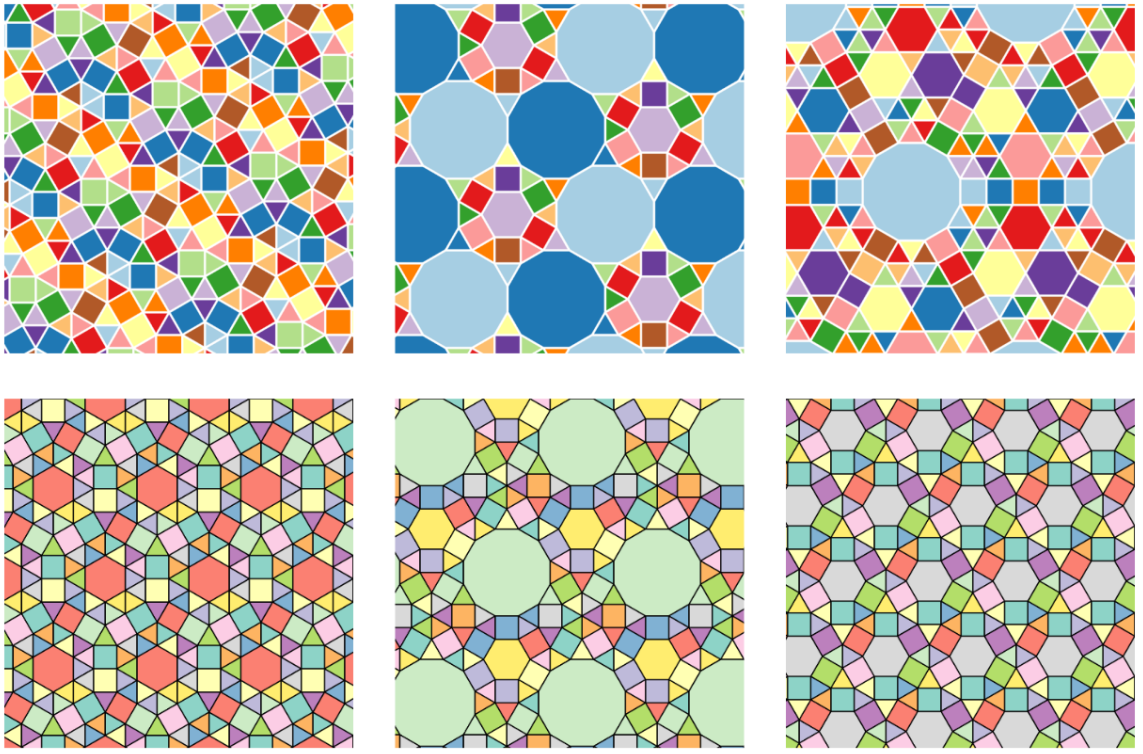
This article is intended to be a temporary 'scrapbook' of how the polykites were conceived. It will contain many raw images and is likely to look messy until I get round to producing something more presentable.

The hat and turtle can only tile the infinite plane aperiodically. They do not permit periodic tilings. They should be treated with respect!

The shaded areas below indicate the similarities between the two polykites. The hat is composed of 8 kites and the turtle, 10.

¿Qué es una teselación (*tiling*)?

- Conjunto de teselas.
- Arreglo de isometrías de las teselas, tal que:
  - Su unión cubre todo el plano.
  - La intersección de su interior es vacía.

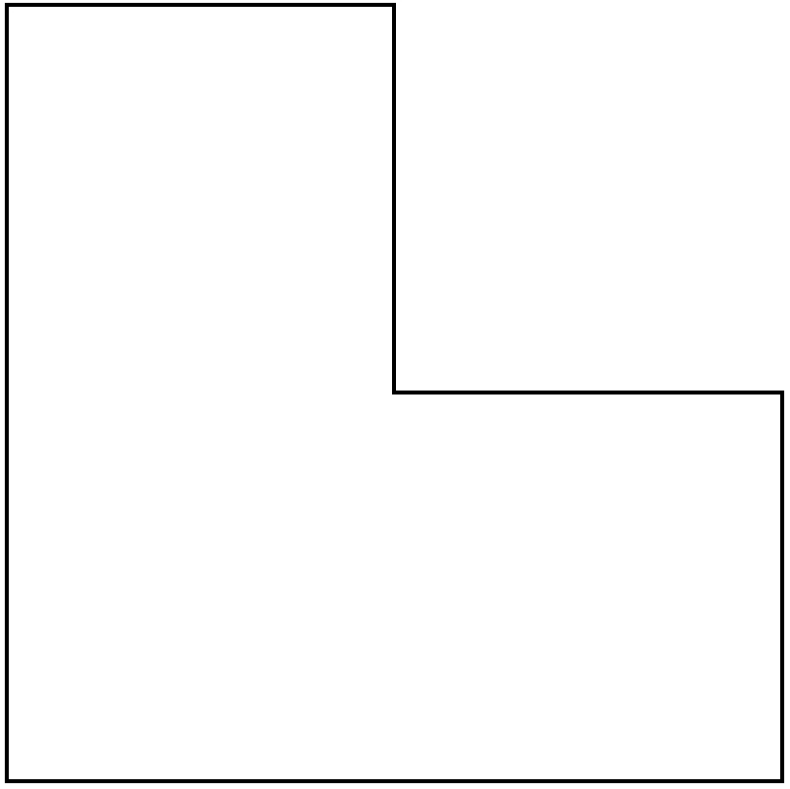


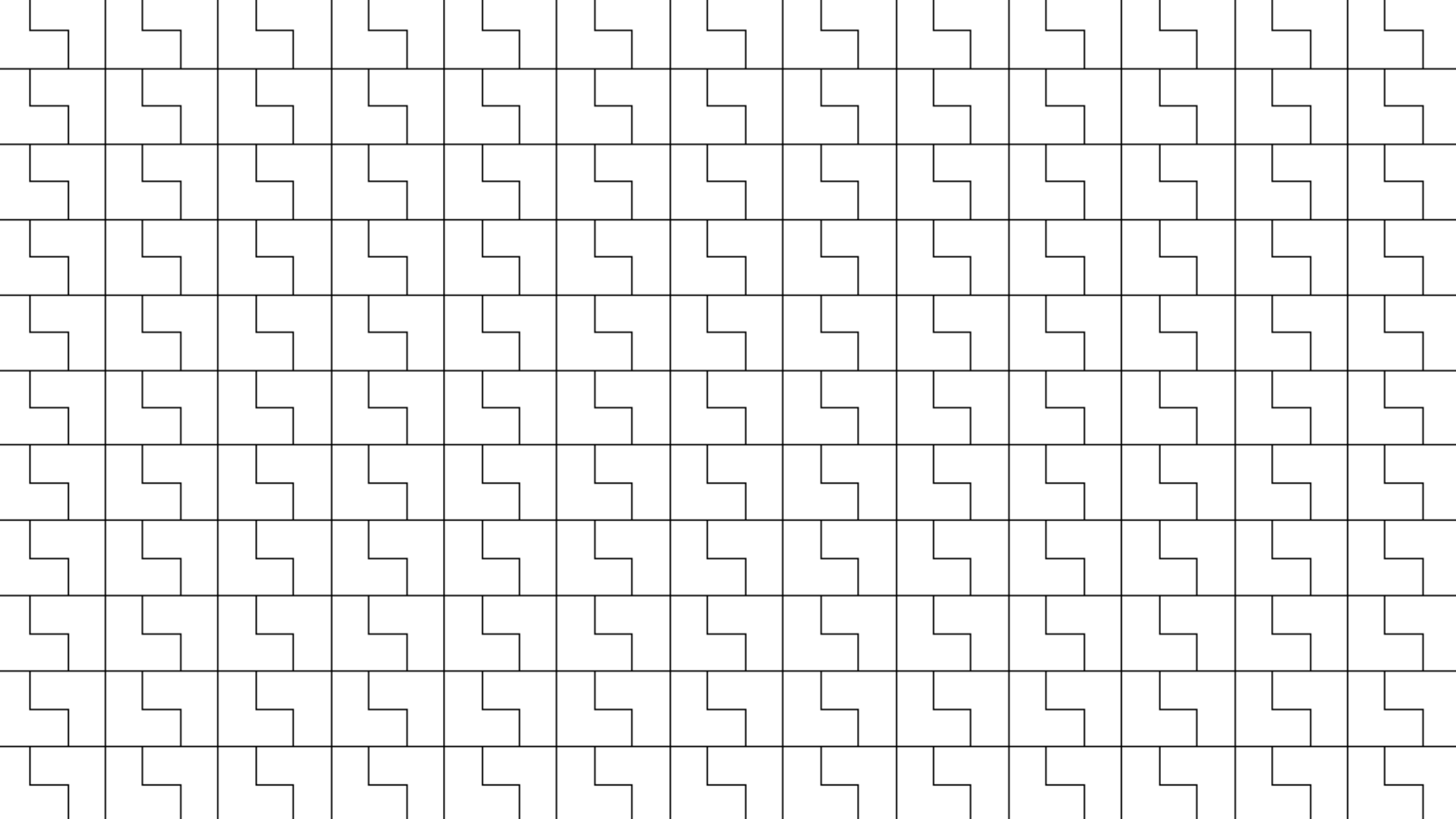
Algo de teoría

- Si existe una teselación con un conjunto ***T*** de teselas, se dice que ***T*** **admite** esa teselación.

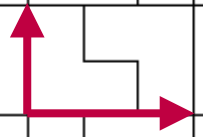
- Si ***k*** teselas son utilizadas para formar una teselación, ésta se llama ***k*-édrica** (*k-hedral*).

Monotile
(monoédrica)



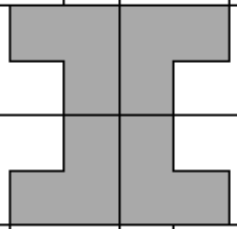


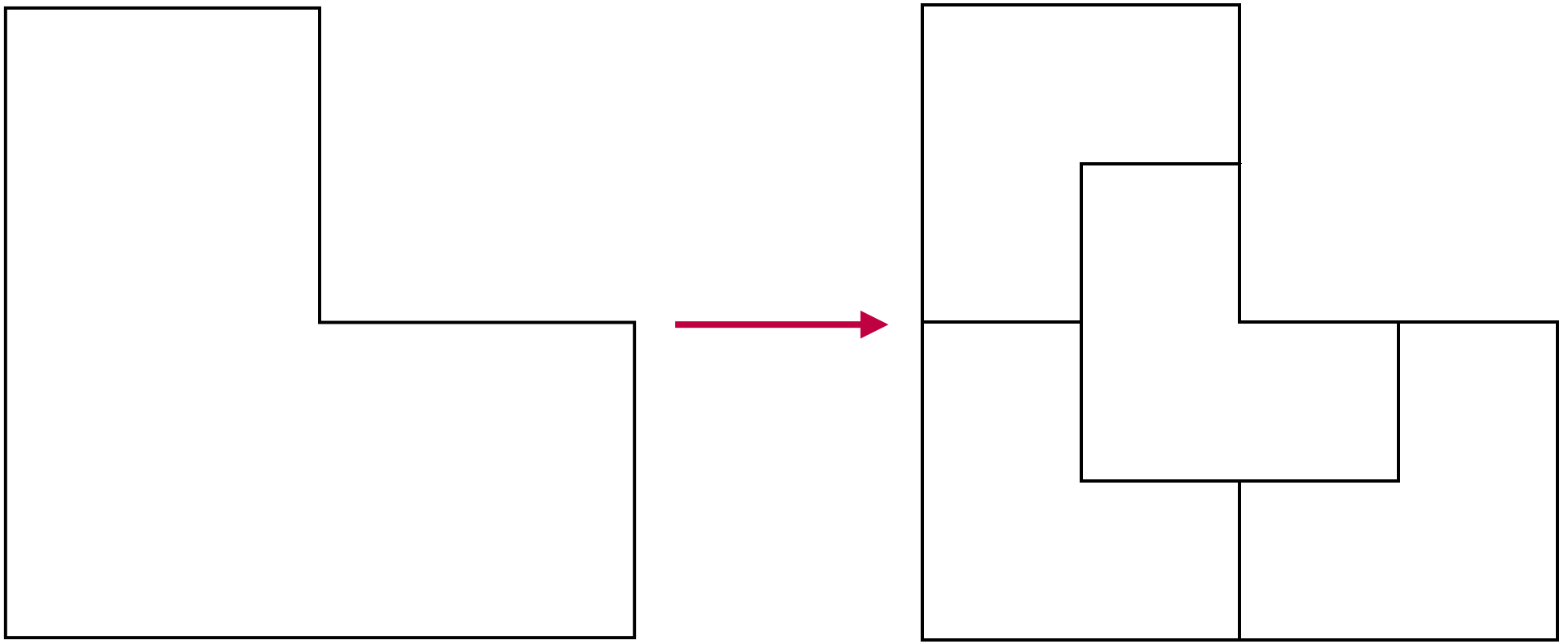
Periódica!

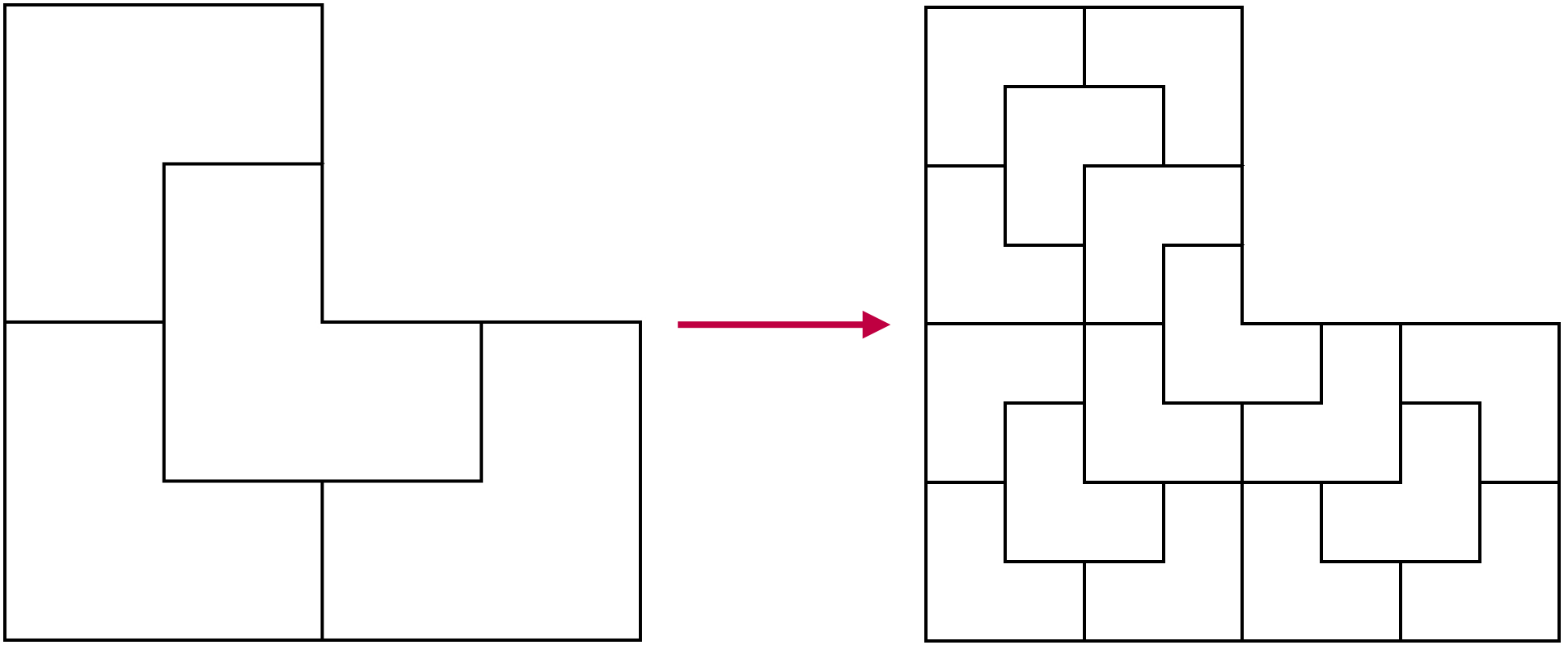


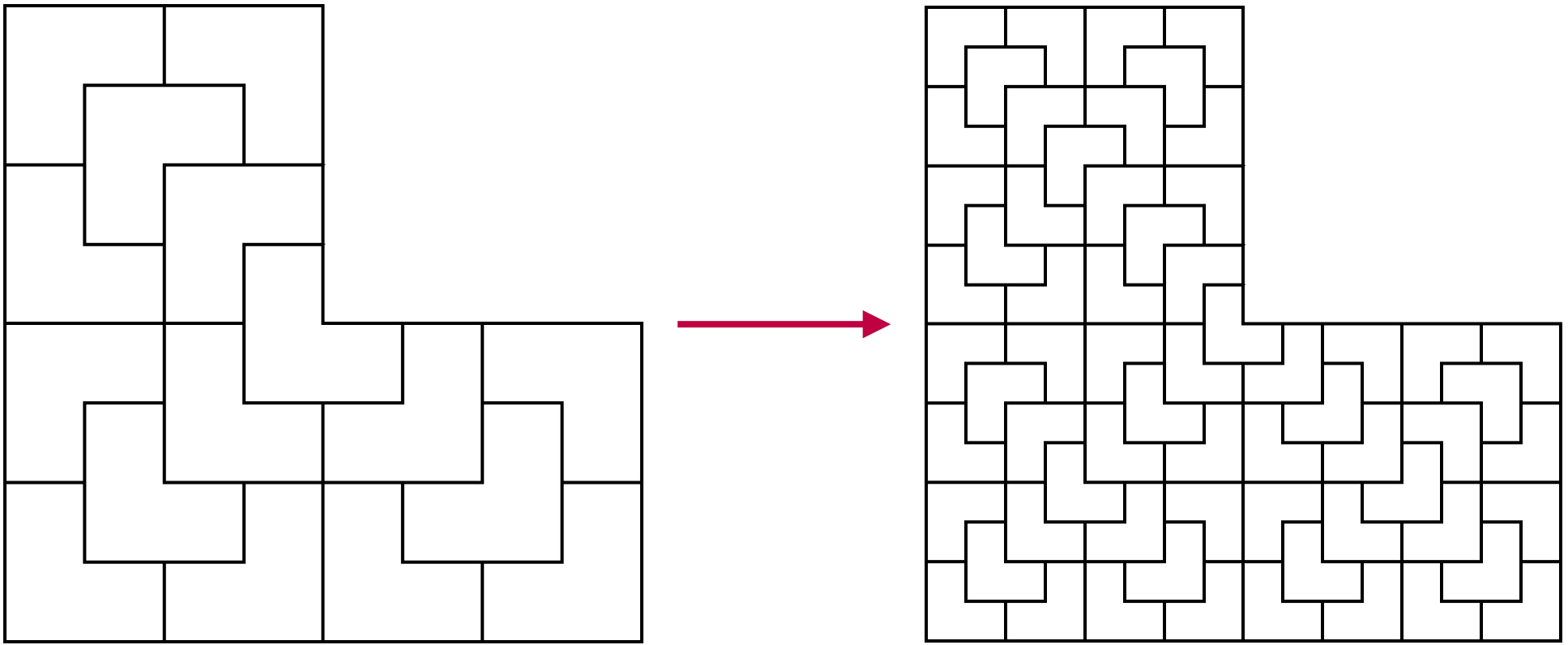

No periódica!

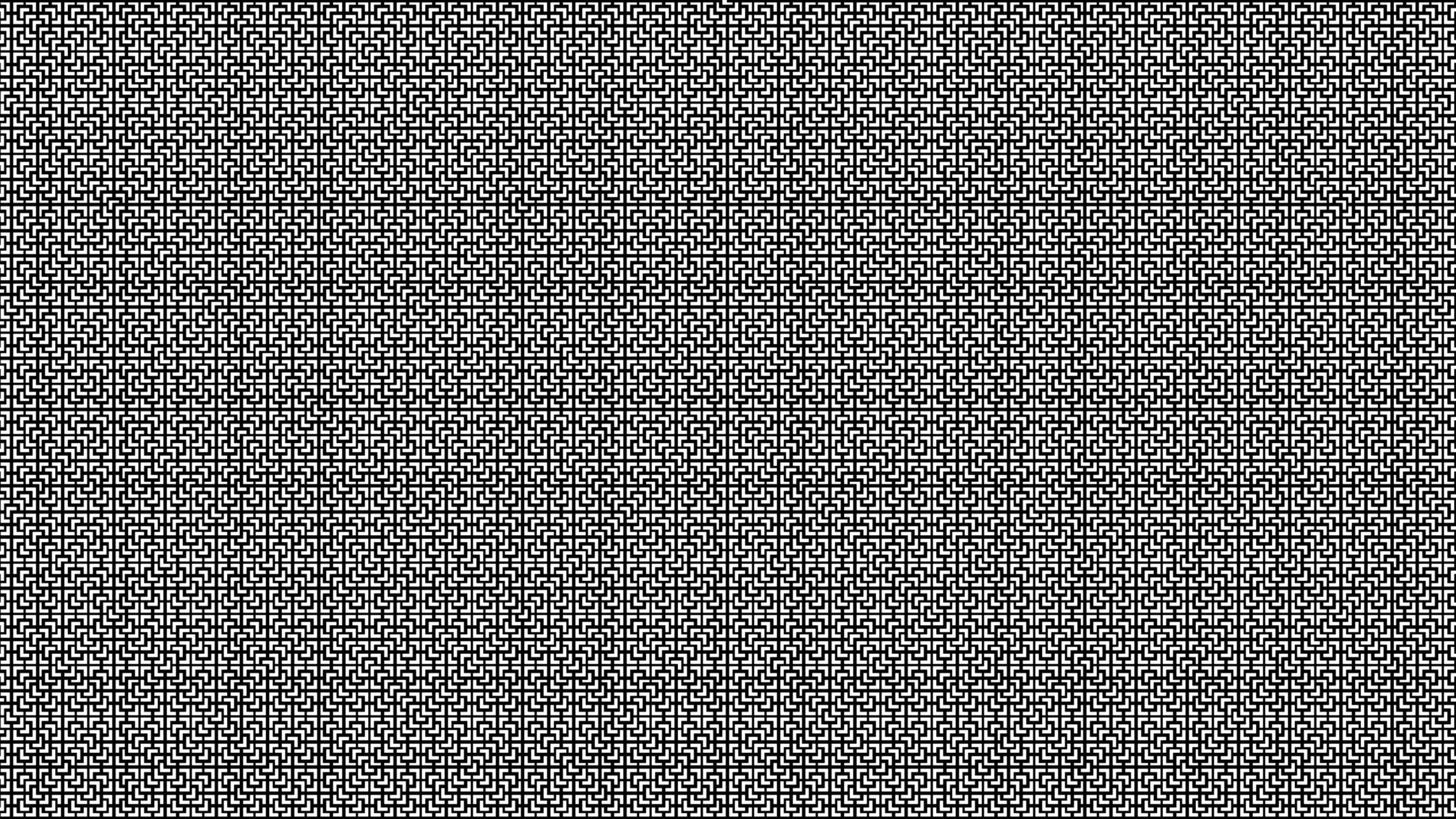
No periódica ≠ aperiódica











Esta sí es aperiódica...
(Chair Tiling)

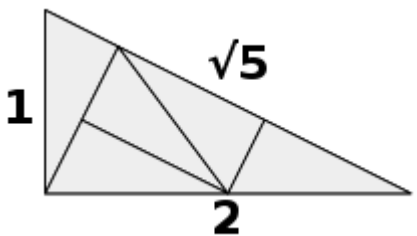
Aperiodicidad

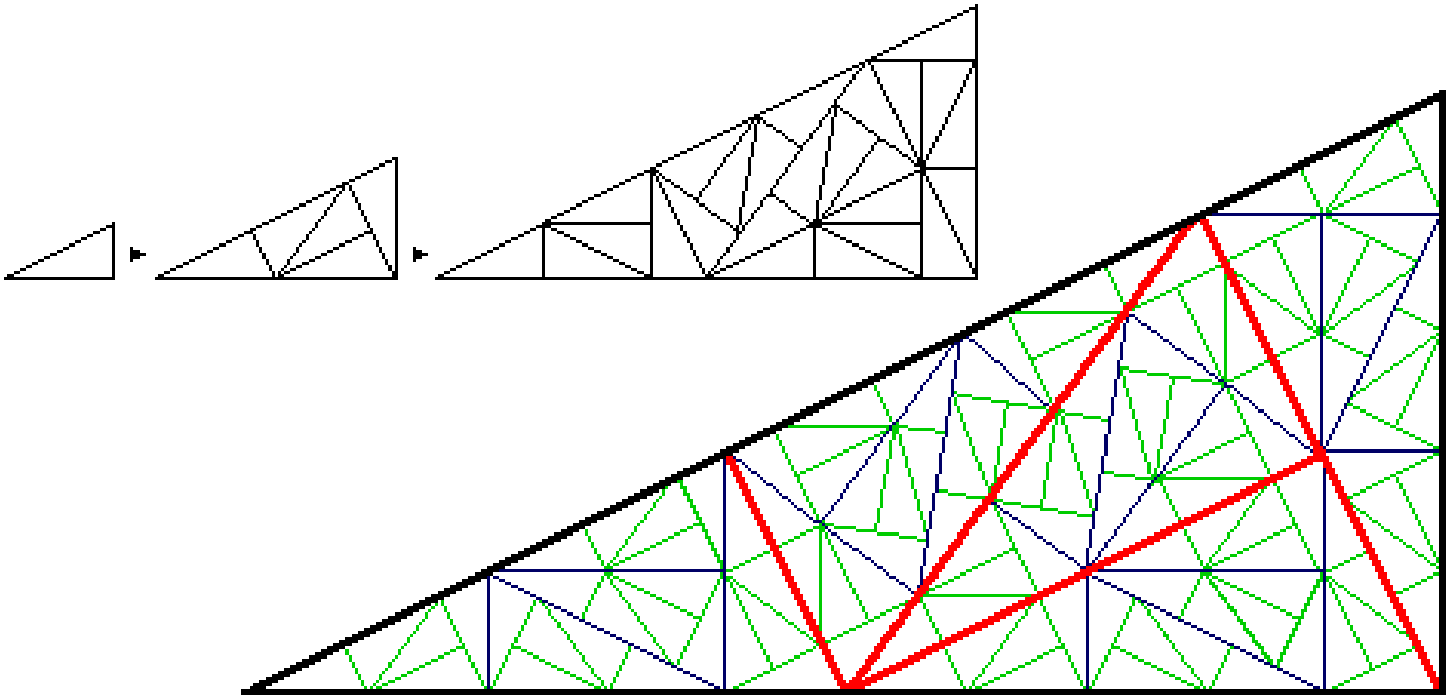
- Teselaciones que no tienen áreas arbitrariamente grandes repetidas.

- Berger (1966) construyó la primera teselación aperiódica usando 20,246 teselas.

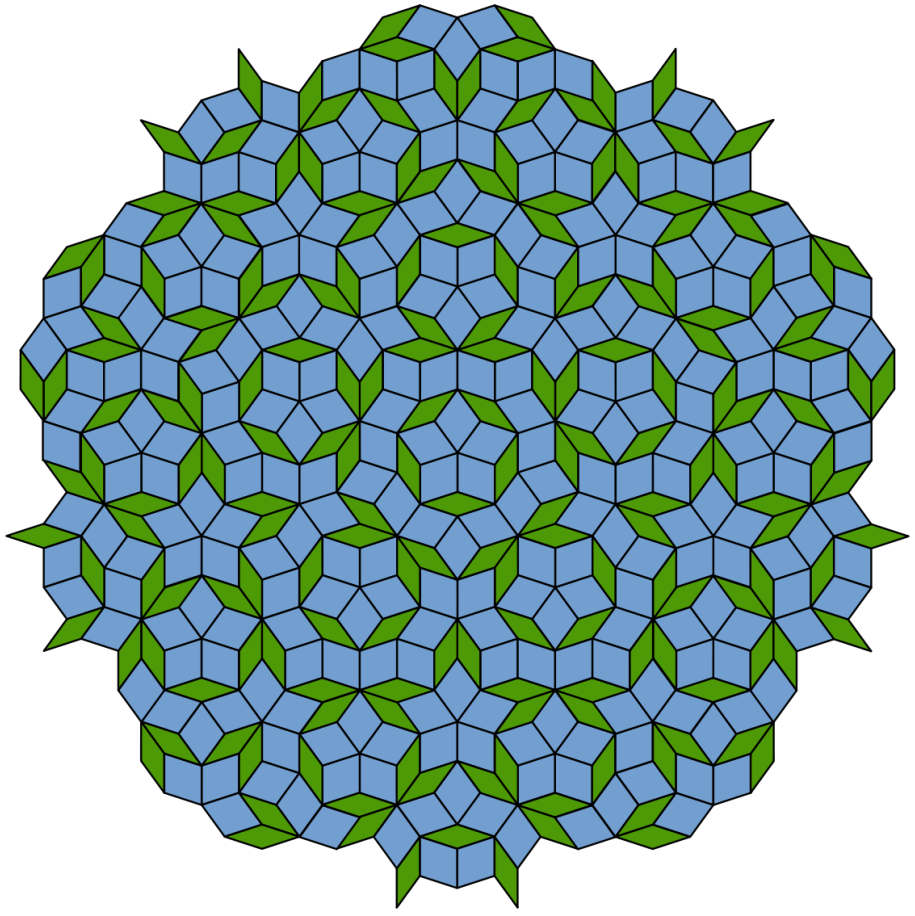
- Aperiódicas famosas...

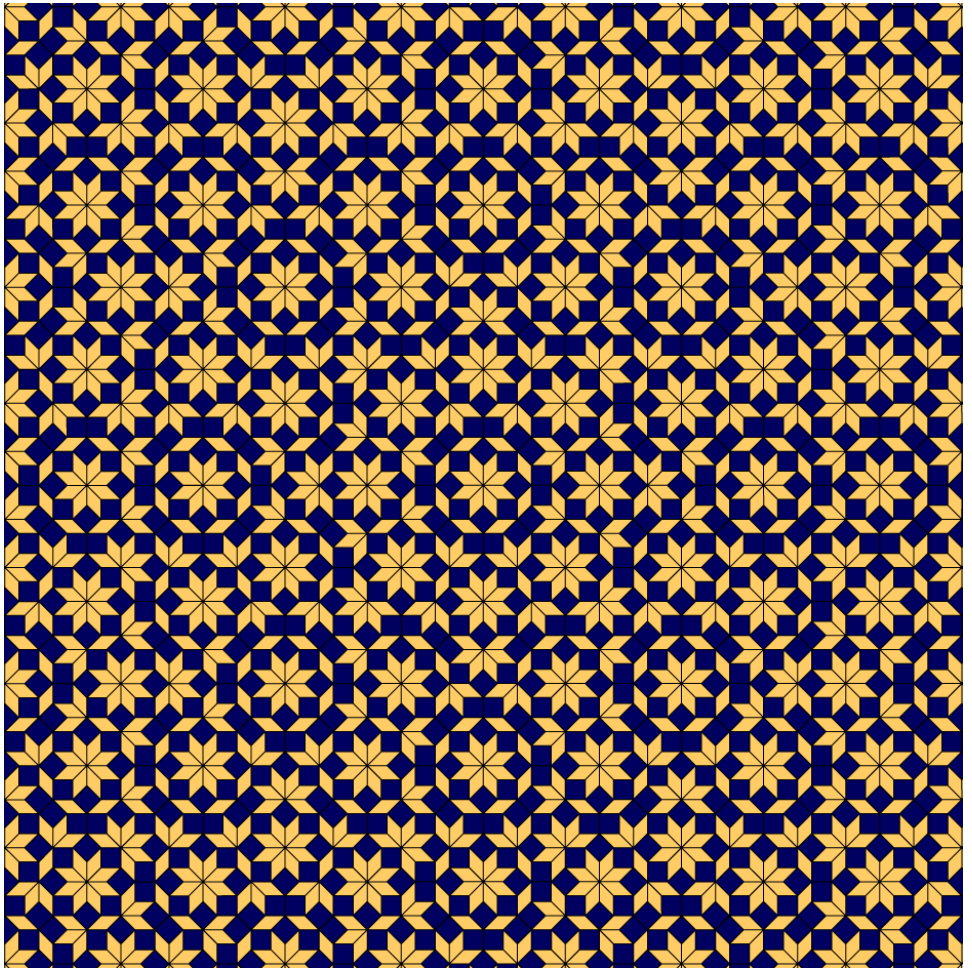
Pinwheel Tiling (Radin & Conway)



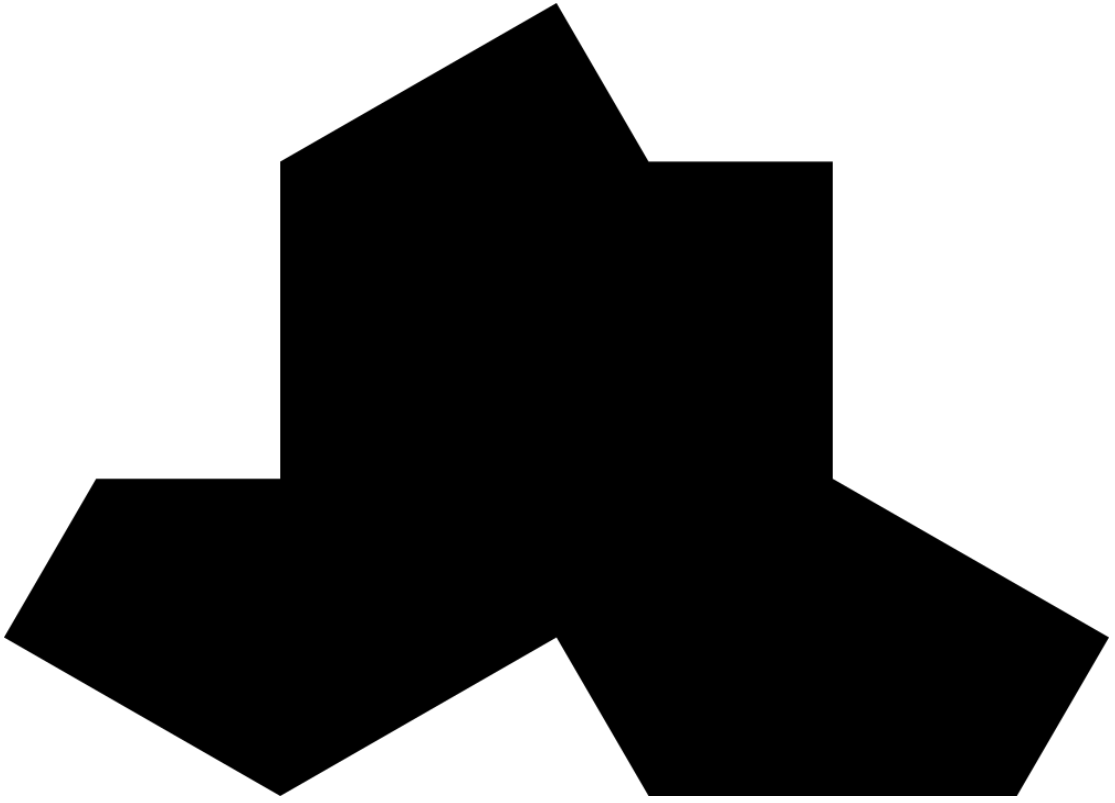


Penrose / Ammann-Beenker





Y entonces… ¿Por qué el alboroto?



Porque tanto la silla,

como el pinwheel

admiten teselaciones periódicas!

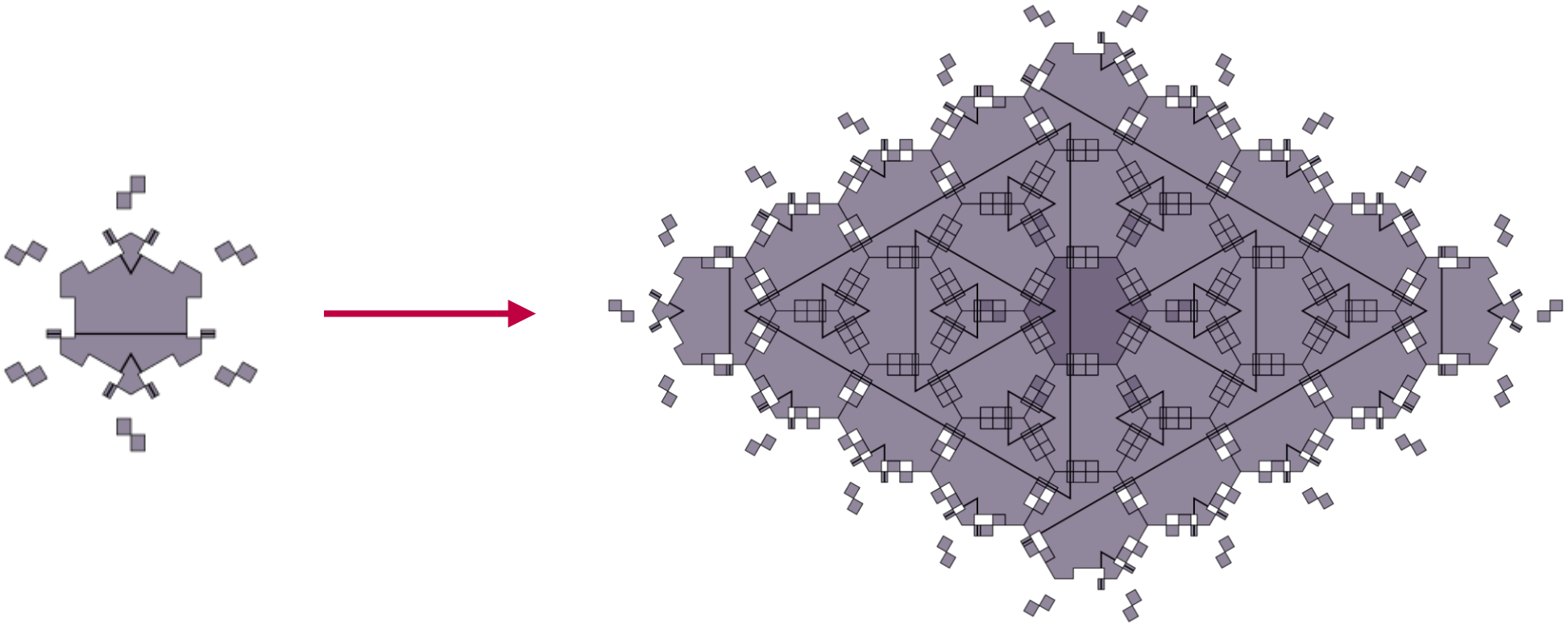
Y la pregunta era:

¿Acaso existe una tesela

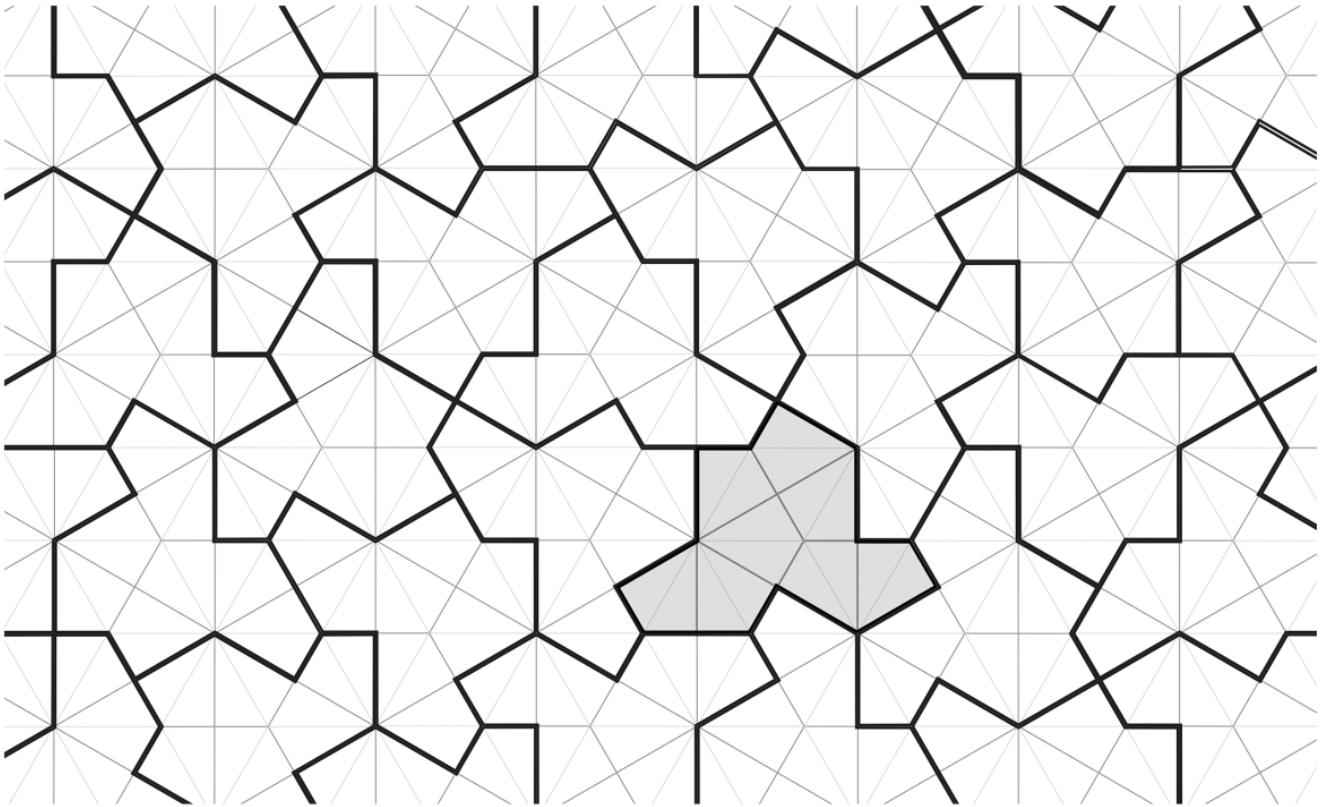
(*ein-stein* = una-piedra) que sólo

admite teselaciones aperiódicas?

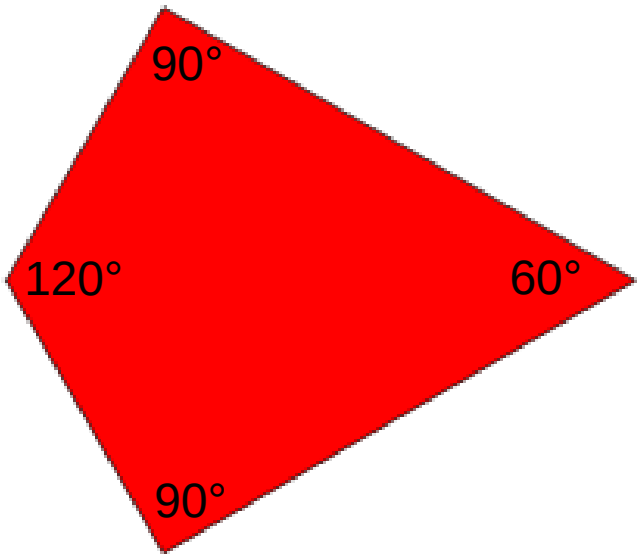
Socolar & Taylor, 2011

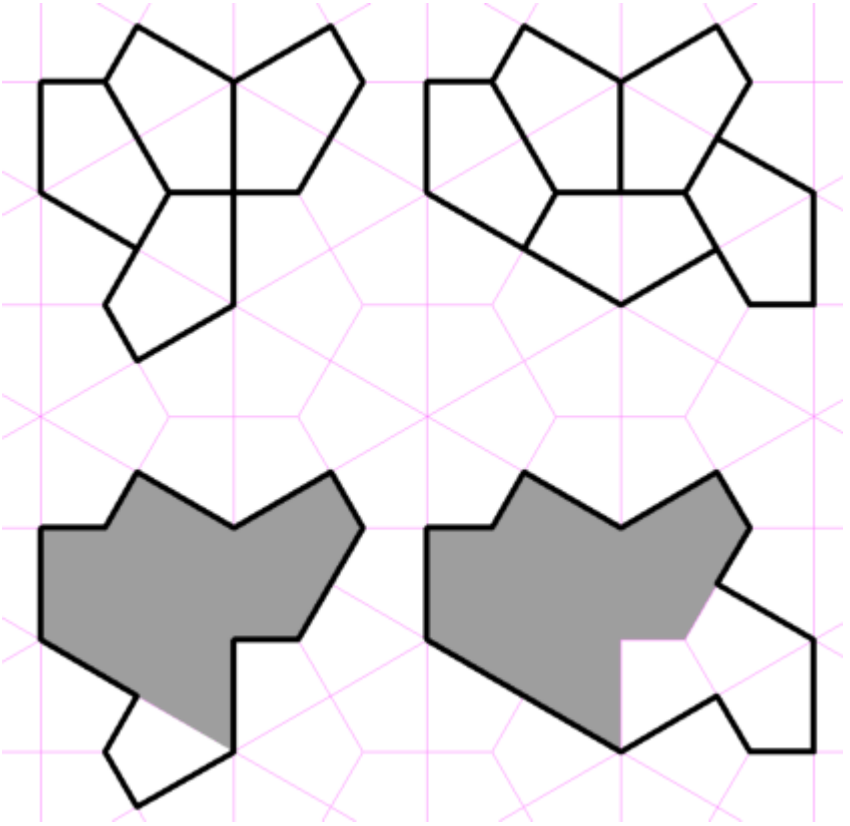


Smith, Myers, Kaplan & Goodman-Strauss, 2023



Espera, ¿son dos polykites?





Es una familia infinita!

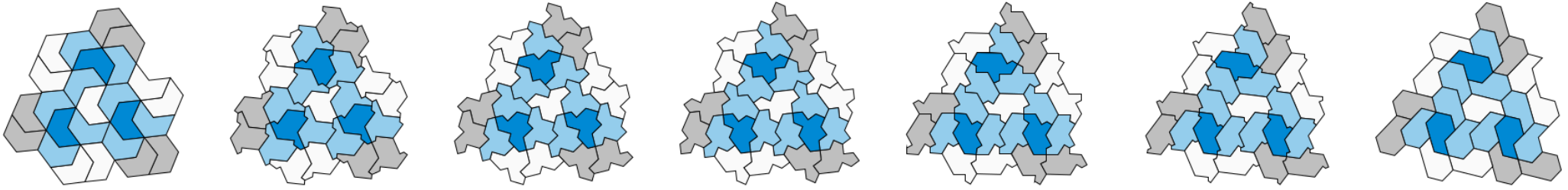
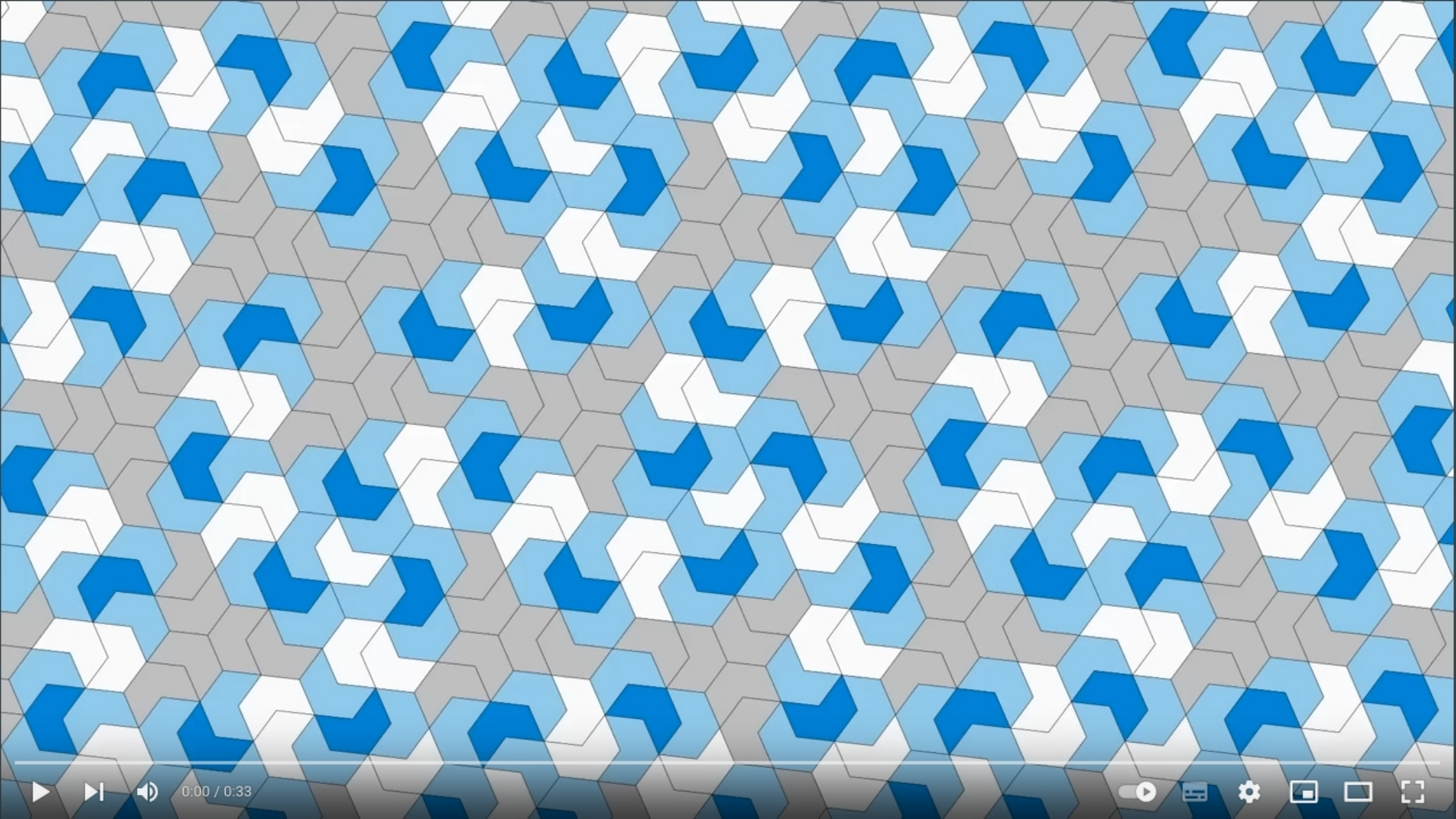
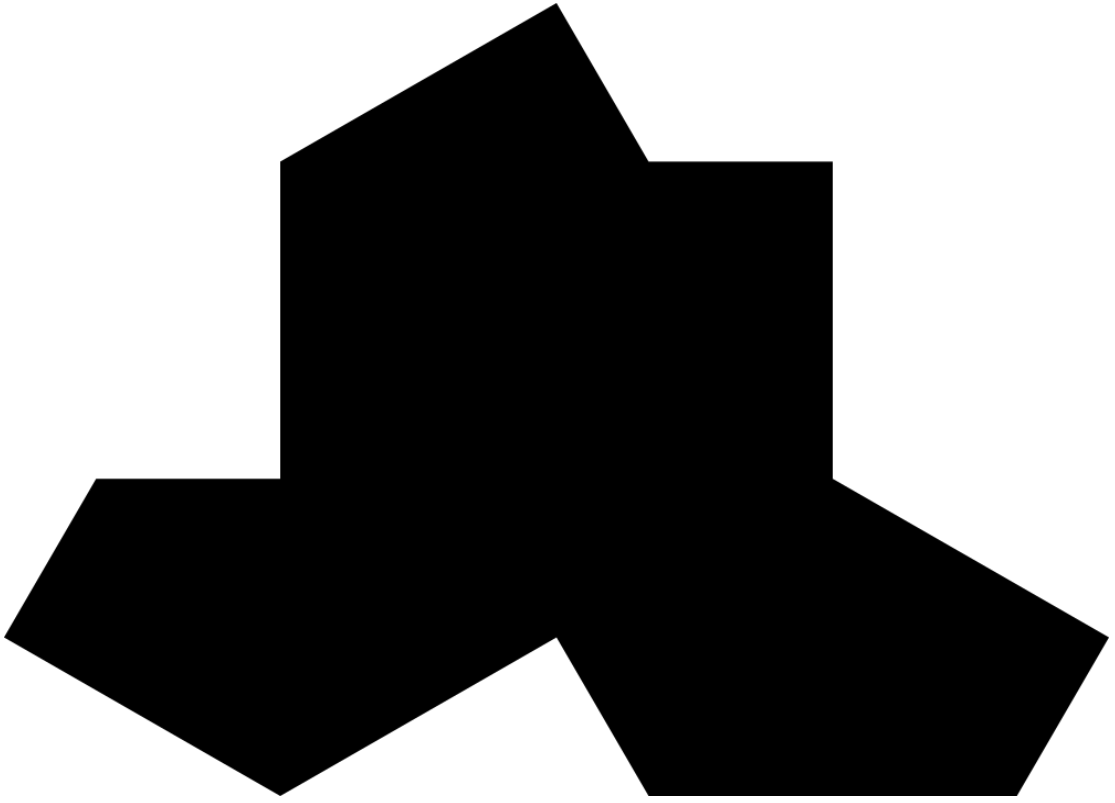


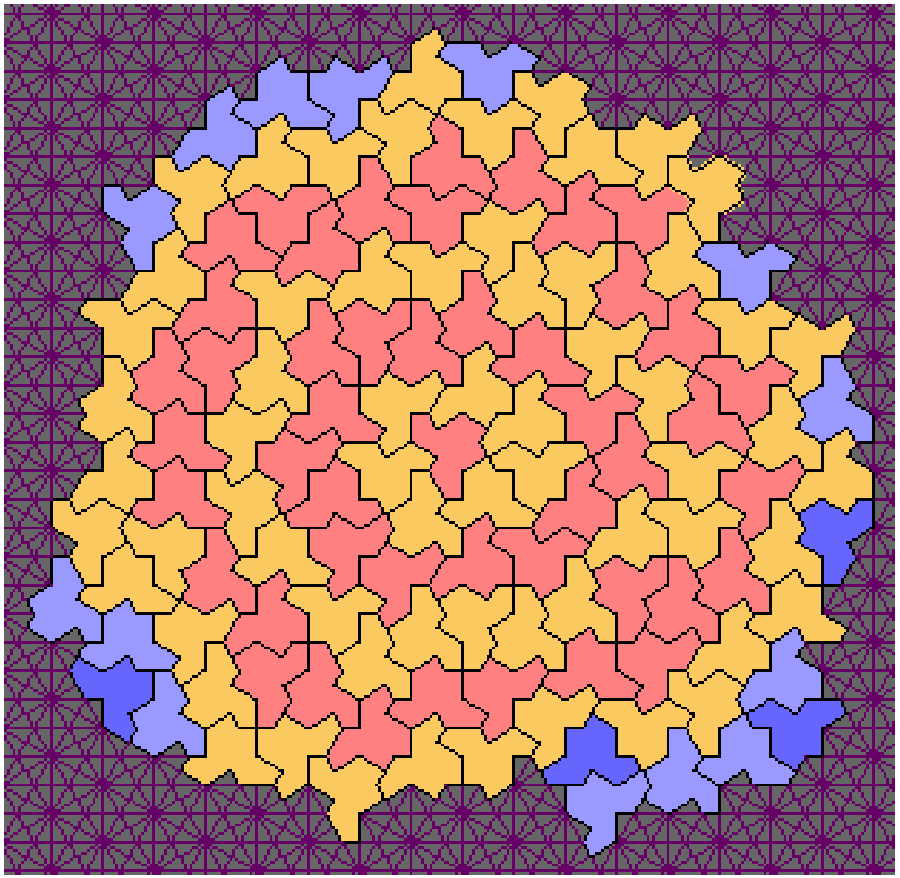
Figure 2.3: The two edge lengths in the hat polykite can be manipulated independently, producing a continuum of shapes. A selection of those shapes is shown here, normalized for scale. $\mathrm{Tile}(0,1)$, $\mathrm{Tile}(1,1)$, and $\mathrm{Tile}(1,0)$ admit periodic tilings; all others are aperiodic.

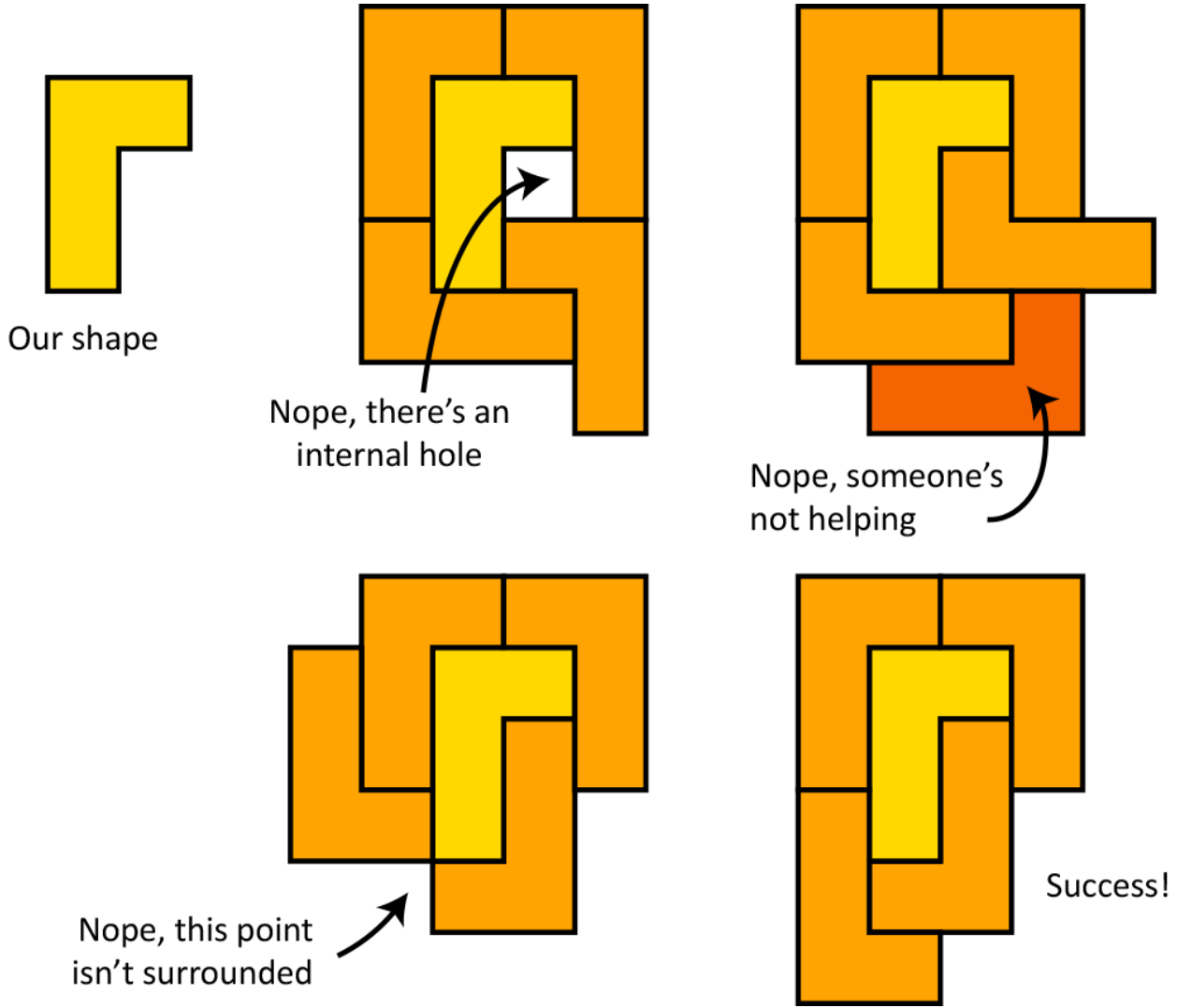

0:00 / 0:33

Y, ¿cómo sabemos que tesela?



Coronas y números de Heesch





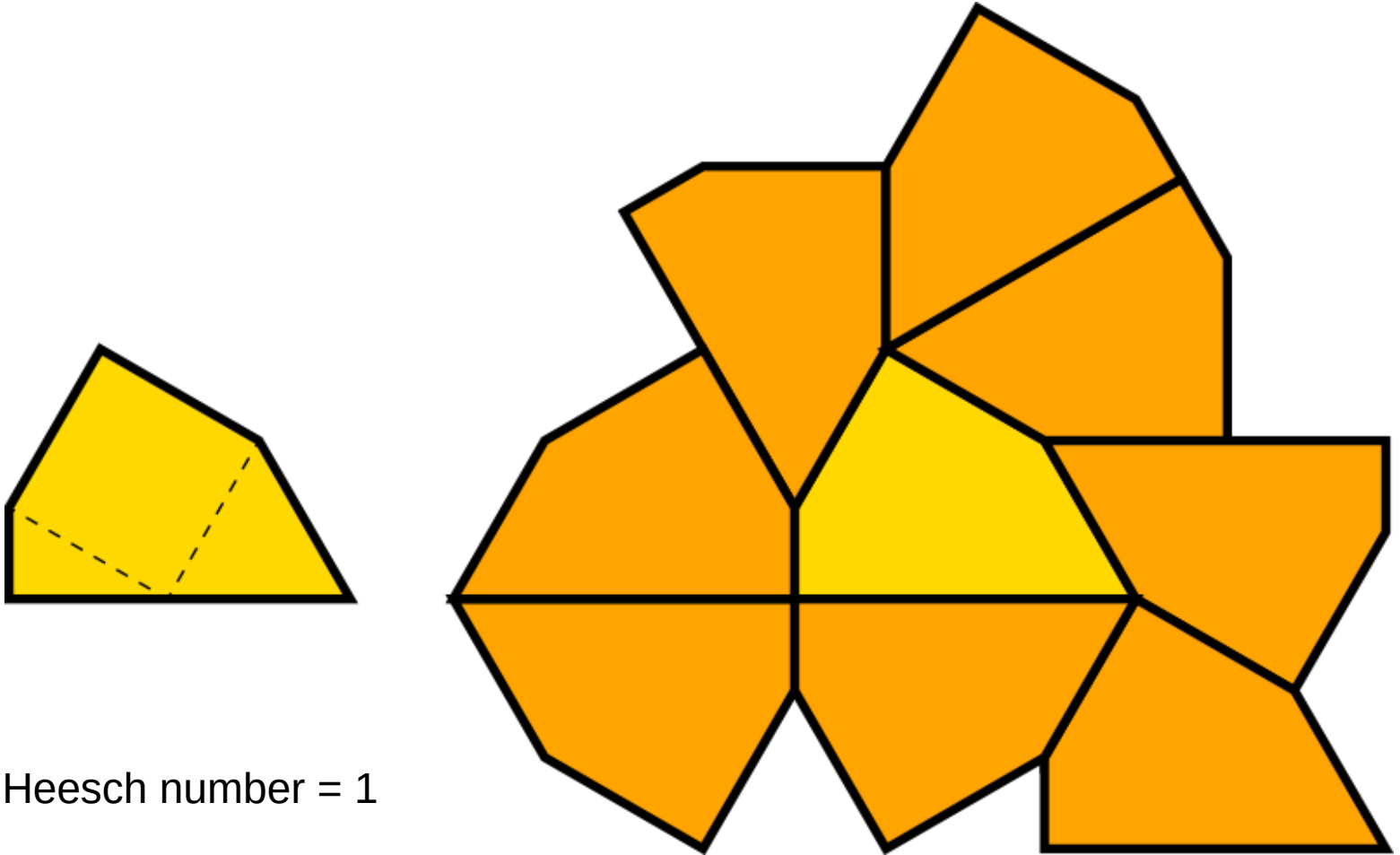
Craig Kaplan
https://isohedral.ca/heesch-numbers-part-1/

Número de Heesch

- Una tesela tiene número de Heesch igual a ***k***, si y sólo si la ***k***-corona es factible (existe una configuración) y la (***k***+1)-corona no lo es.

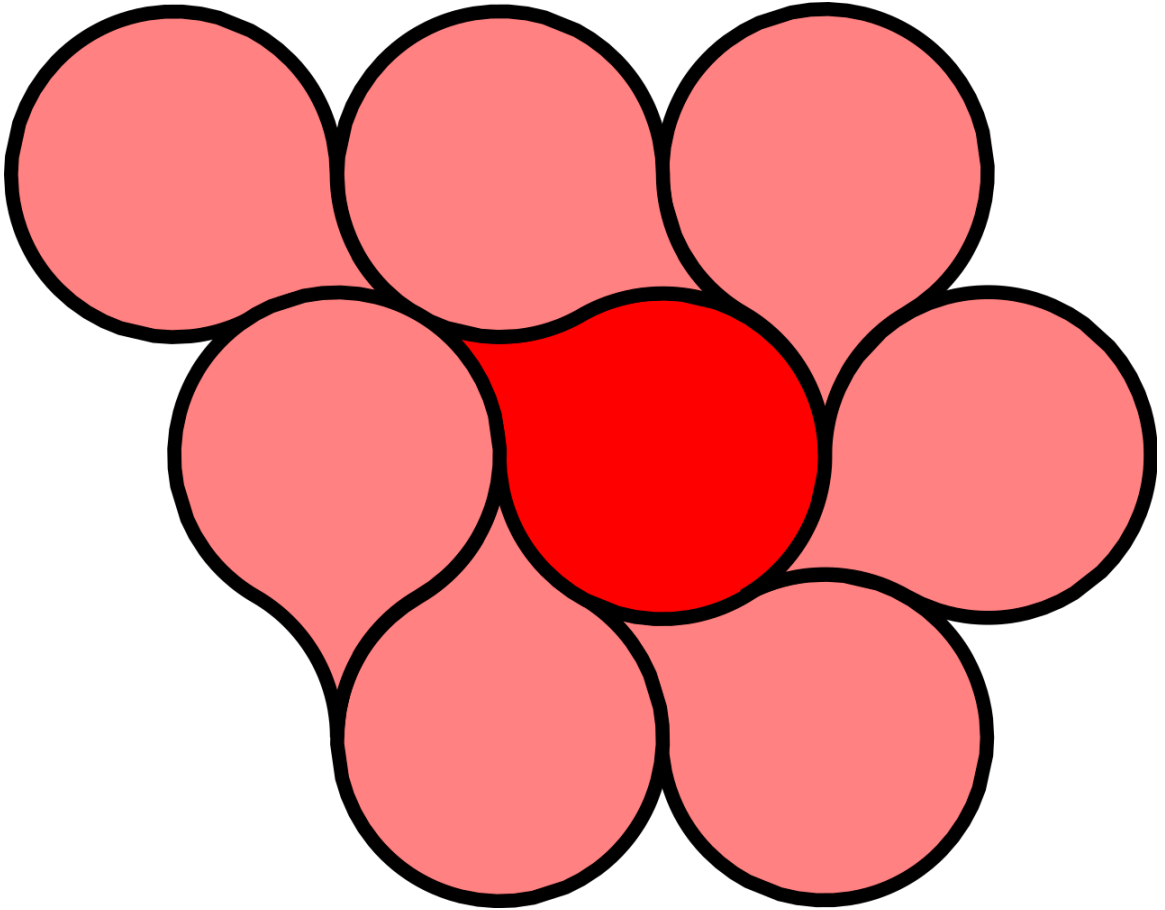
- Por ejemplo:

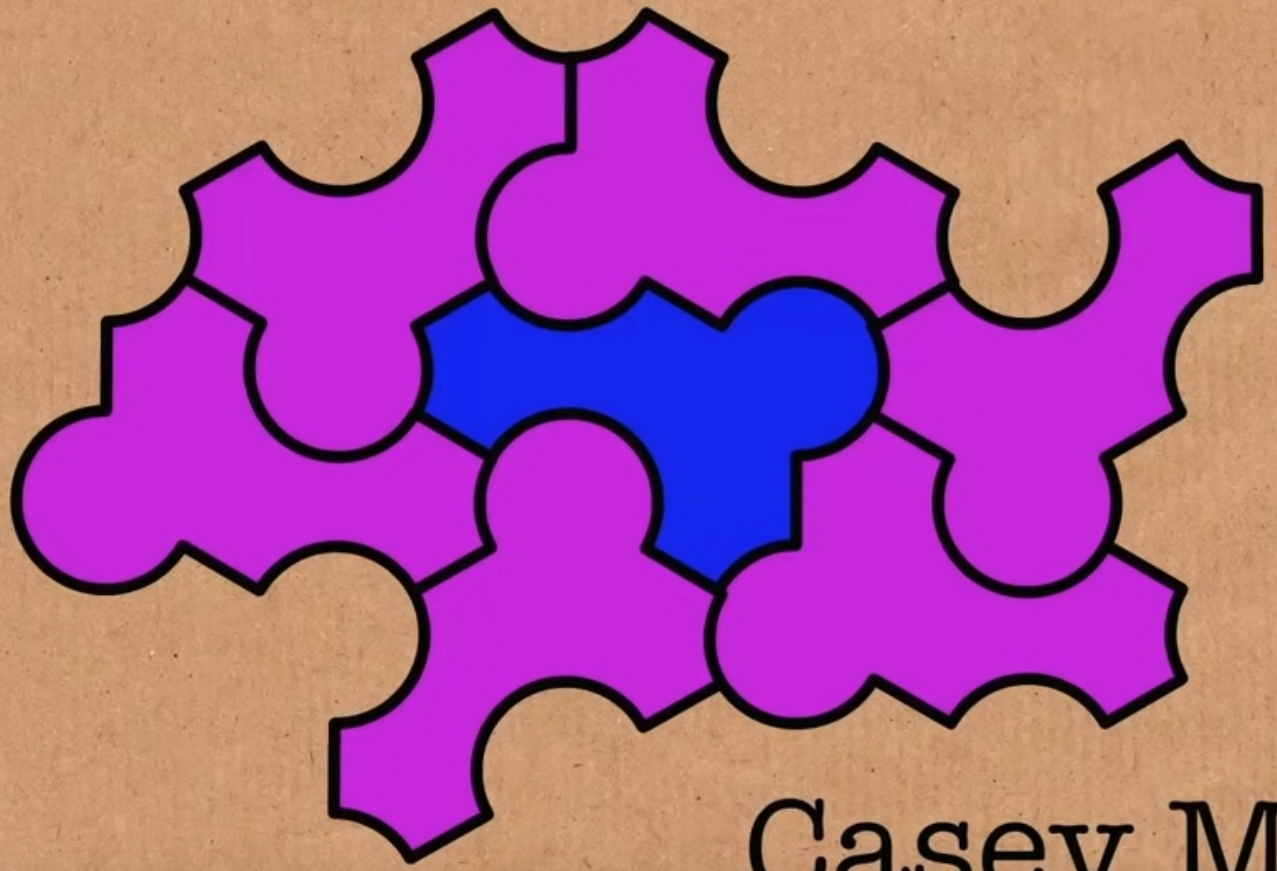
Heesch, 1968



Heesch number = 1

Walther Lietzmann, 1928

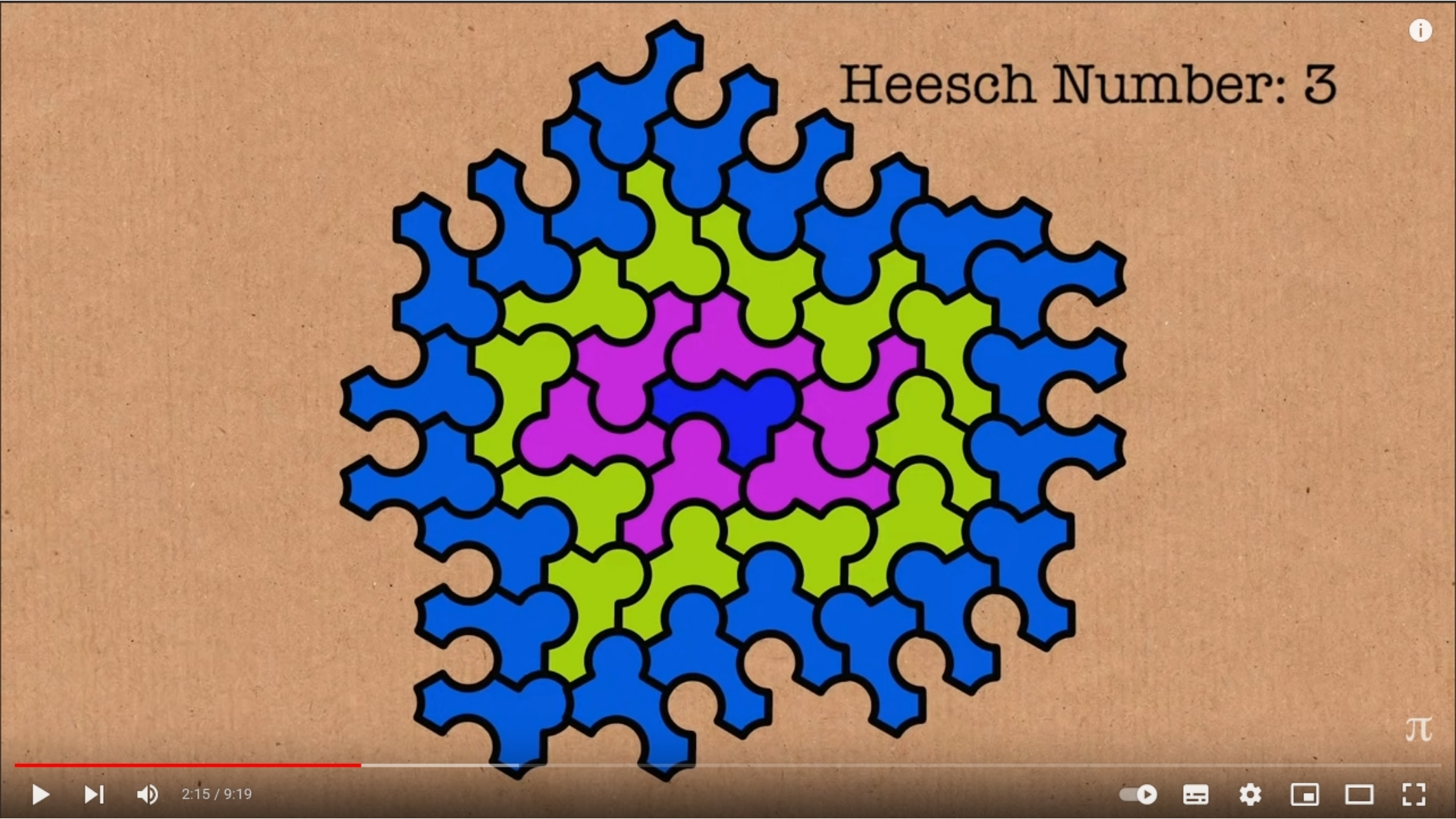


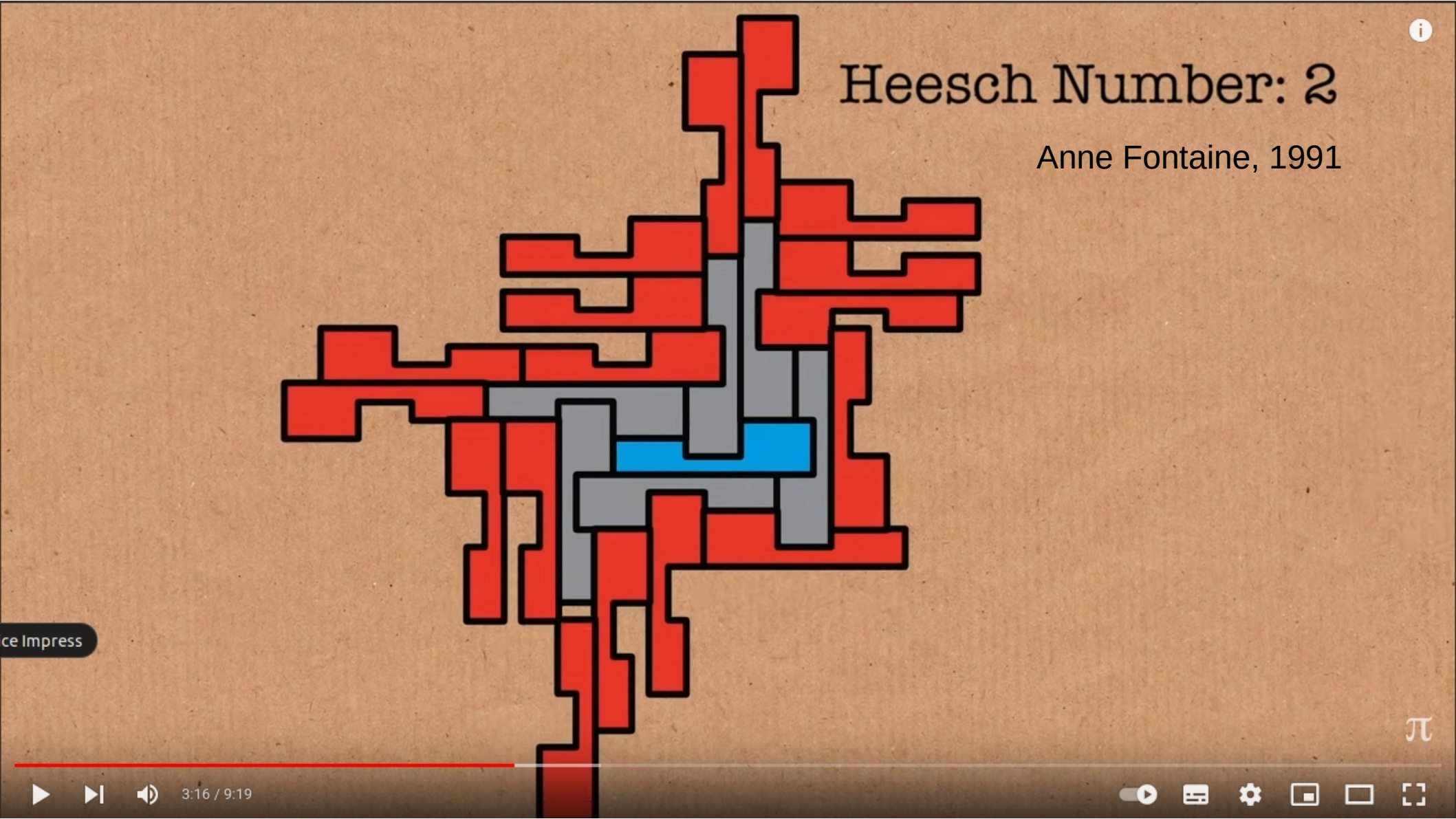


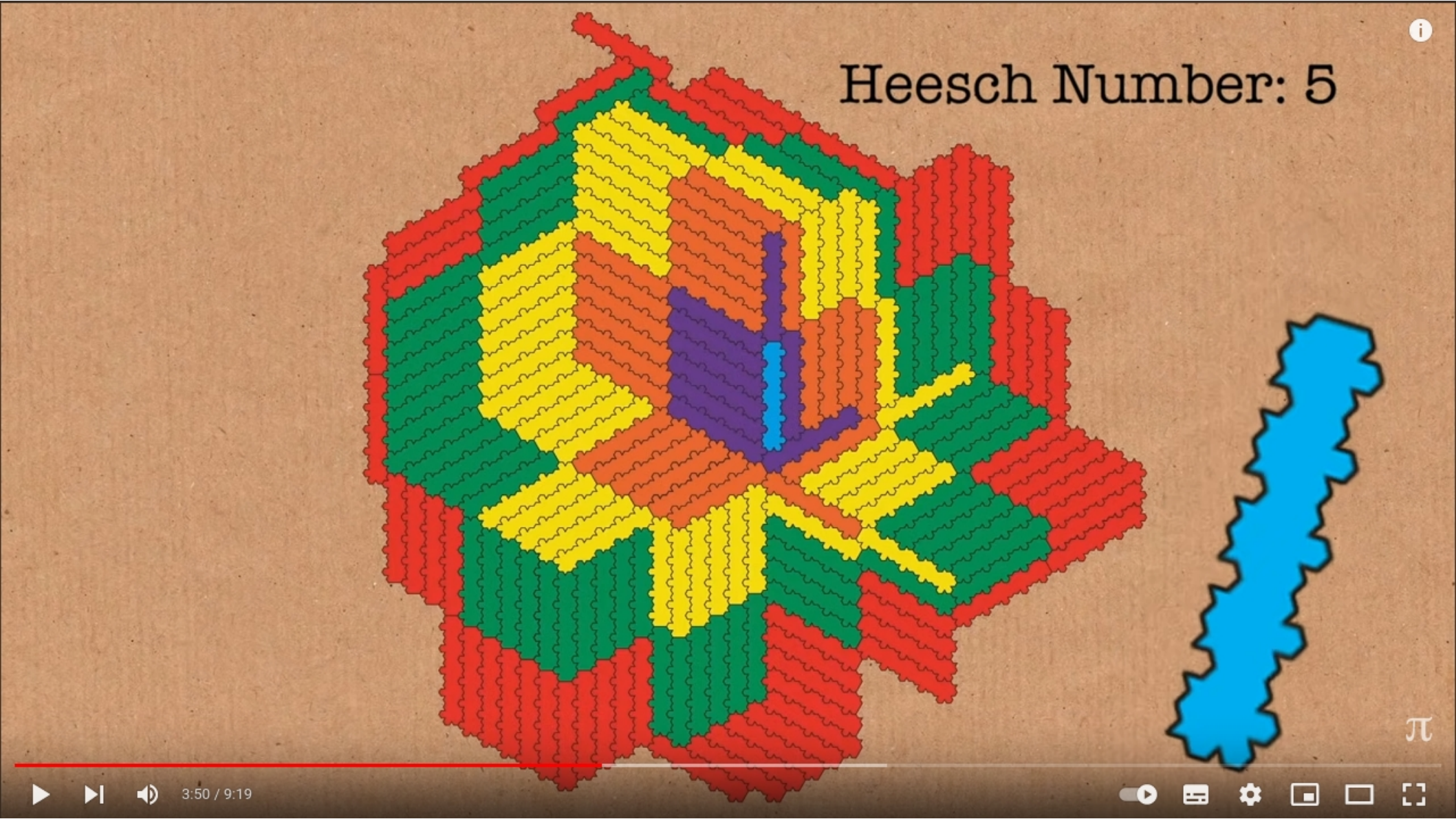
Casey Mann

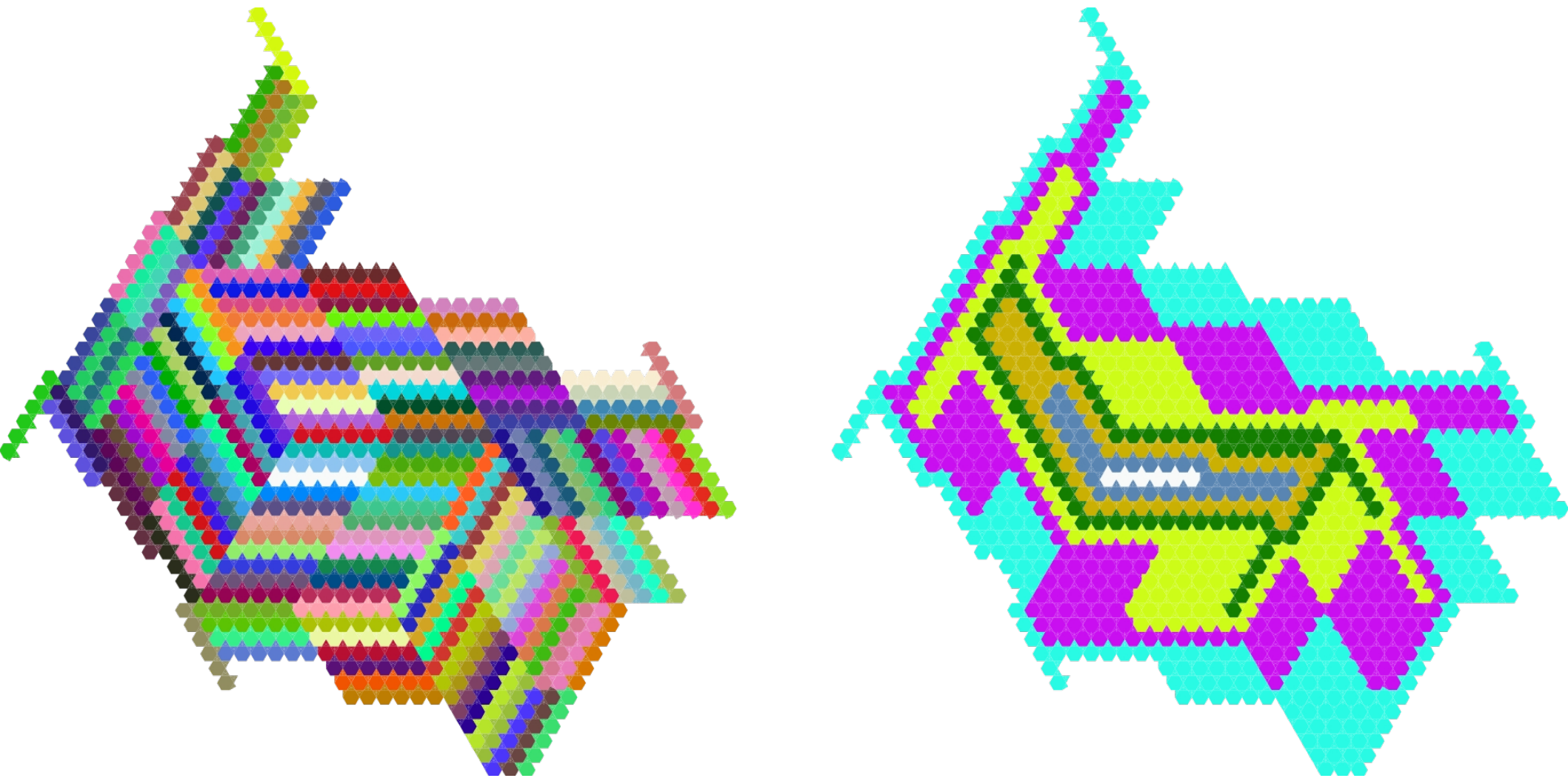
π

0:43 / 9:19


Heesch Number: 3
2:15 / 9:19


Heesch Number: 2
Anne Fontaine, 1991
ce Impress
3:16 / 9:19


Heesch Number: 5
3:50 / 9:19



Bojan Bašić, 2020

Número de Heesch

Obs. Si una tesela admite una teselación, su número de Heesch es igual a ∞.

- ¿Para cualquier entero ***k*** existe una tesela cuyo número de Heesch es igual a ***k*** ?
- ¿Existe ***H*** tal que si el número de Heesch de una tesela es mayor o igual a ***H***, entonces esta tesela admite una teselación?

Aperiodicidad

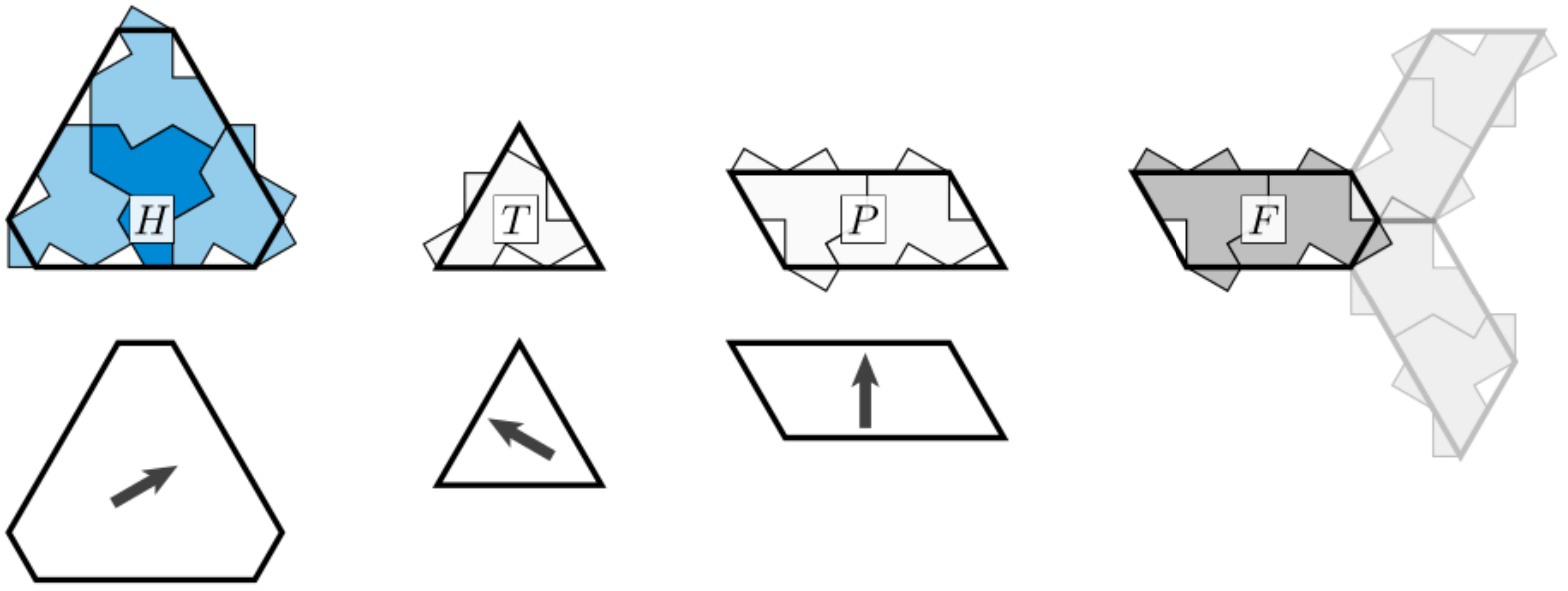


Figure 2.5: The H, T, P, and F metatiles (top), constructed by simplifying the boundaries of clusters of hats. We mark the H, T, and P metatiles with arrows when needed (bottom), to distinguish between otherwise symmetric orientations.

Smith et al. 2023

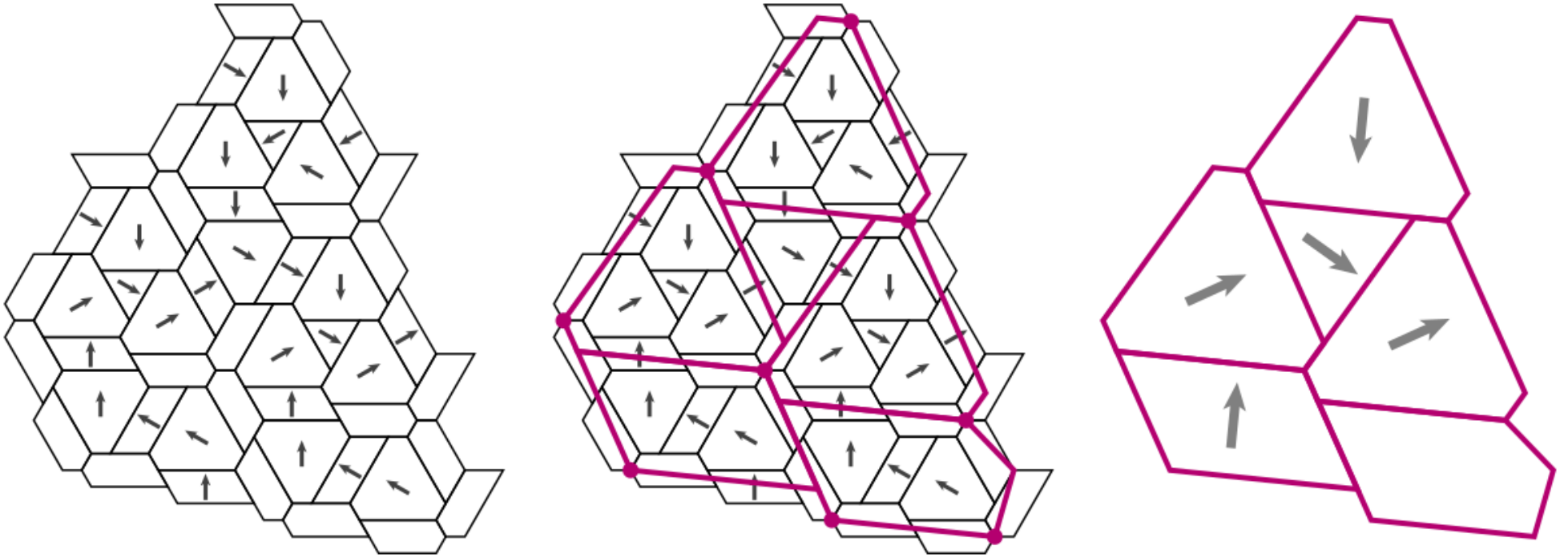


Figure 2.6: The construction of a family of supertiles from a patch of metatiles. The patch of metatiles on the left can be used to locate key vertices of the supertiles, marked with red dots in the central diagram. Those dots, together with constraints on angles, fully determine the shapes of the supertiles, which are not merely scaled-up copies of their progenitors. On the right, the supertiles are marked with arrows indicating their orientations.

Smith et al. 2023

Smith et al. 2023

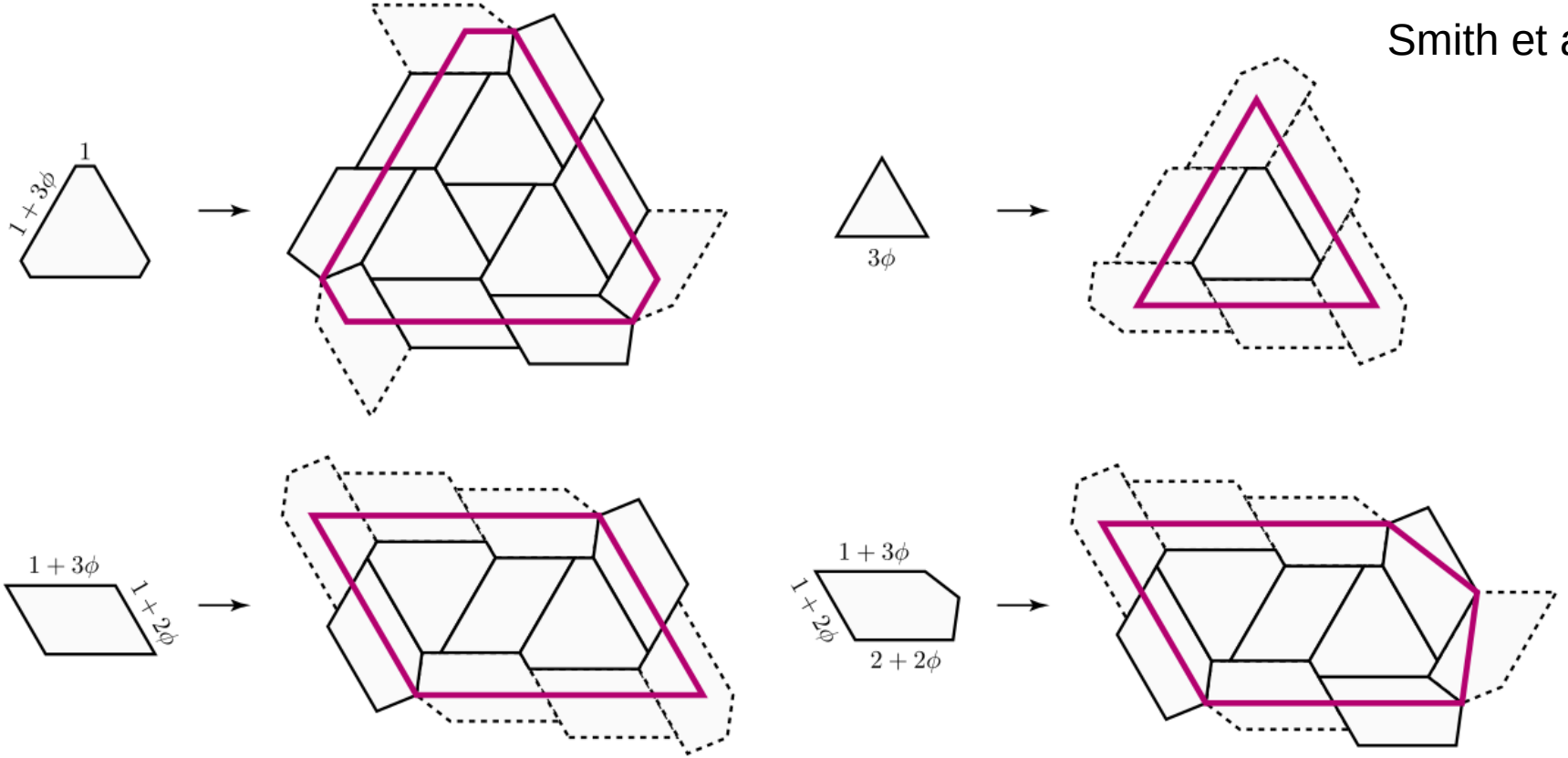
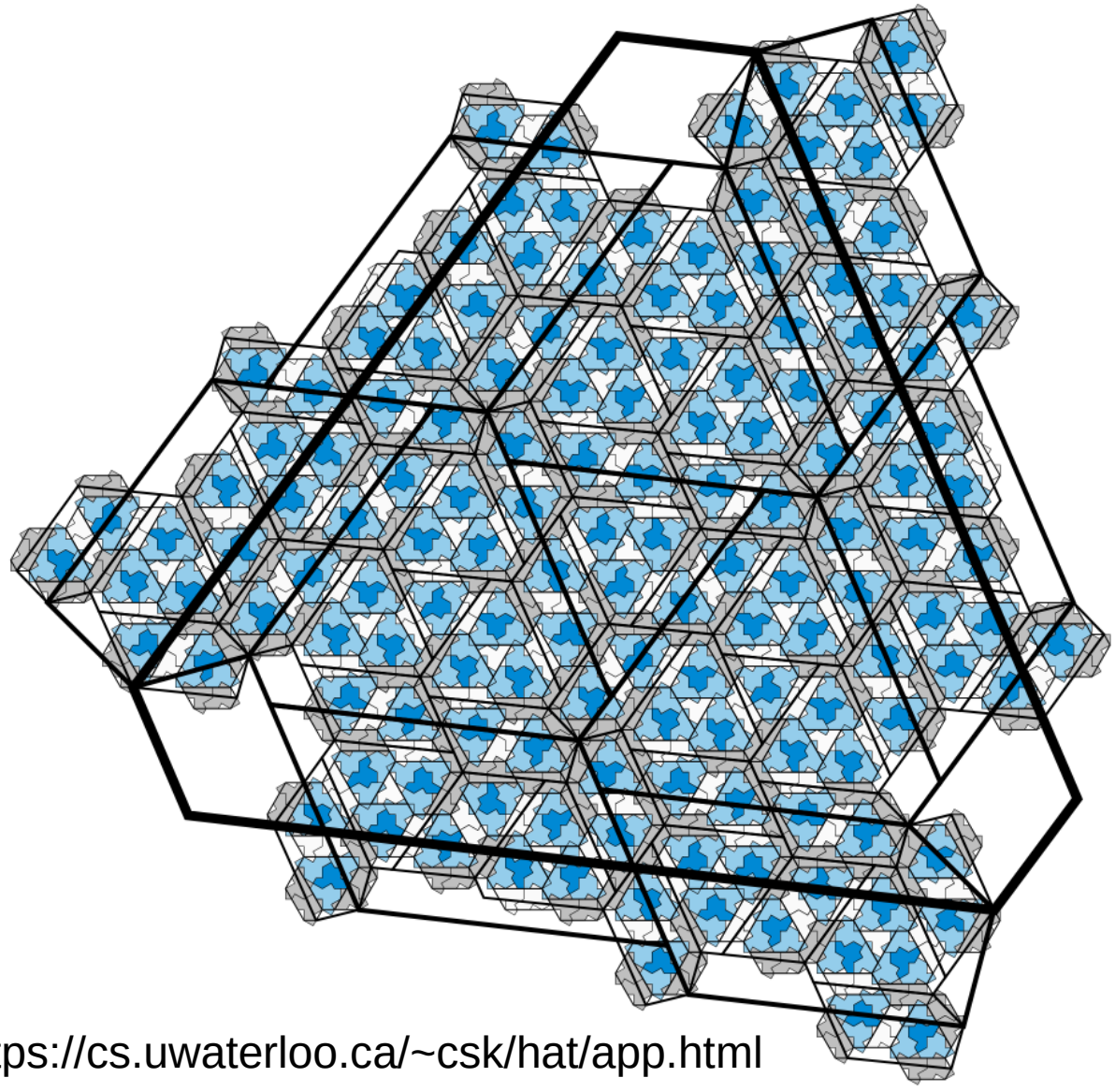
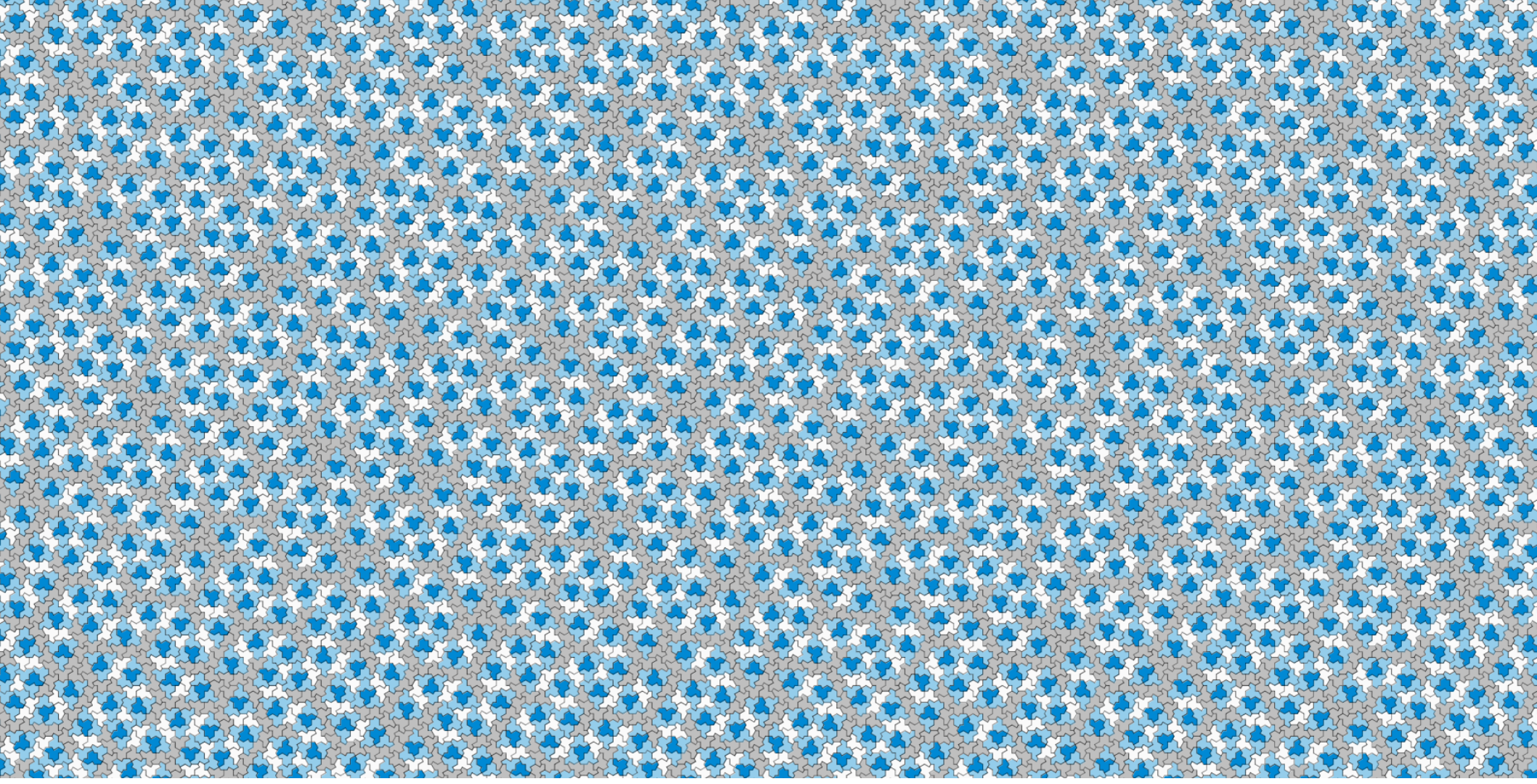


Figure 2.8: A substitution system based on converged tile shapes. Scaling the tiles so that the the short edges of the H tile have unit length, all tile edges except the two adjacent to fylfot centres have lengths in $\mathbb{Z}[\phi]$, where ϕ is the golden ratio. In each substitution rule, tiles shown with dashed boundaries can be omitted, leading to patches in which there are no duplicate tiles contributed by supertiles sharing an edge.



https://cs.uwaterloo.ca/~csk/hat/app.html



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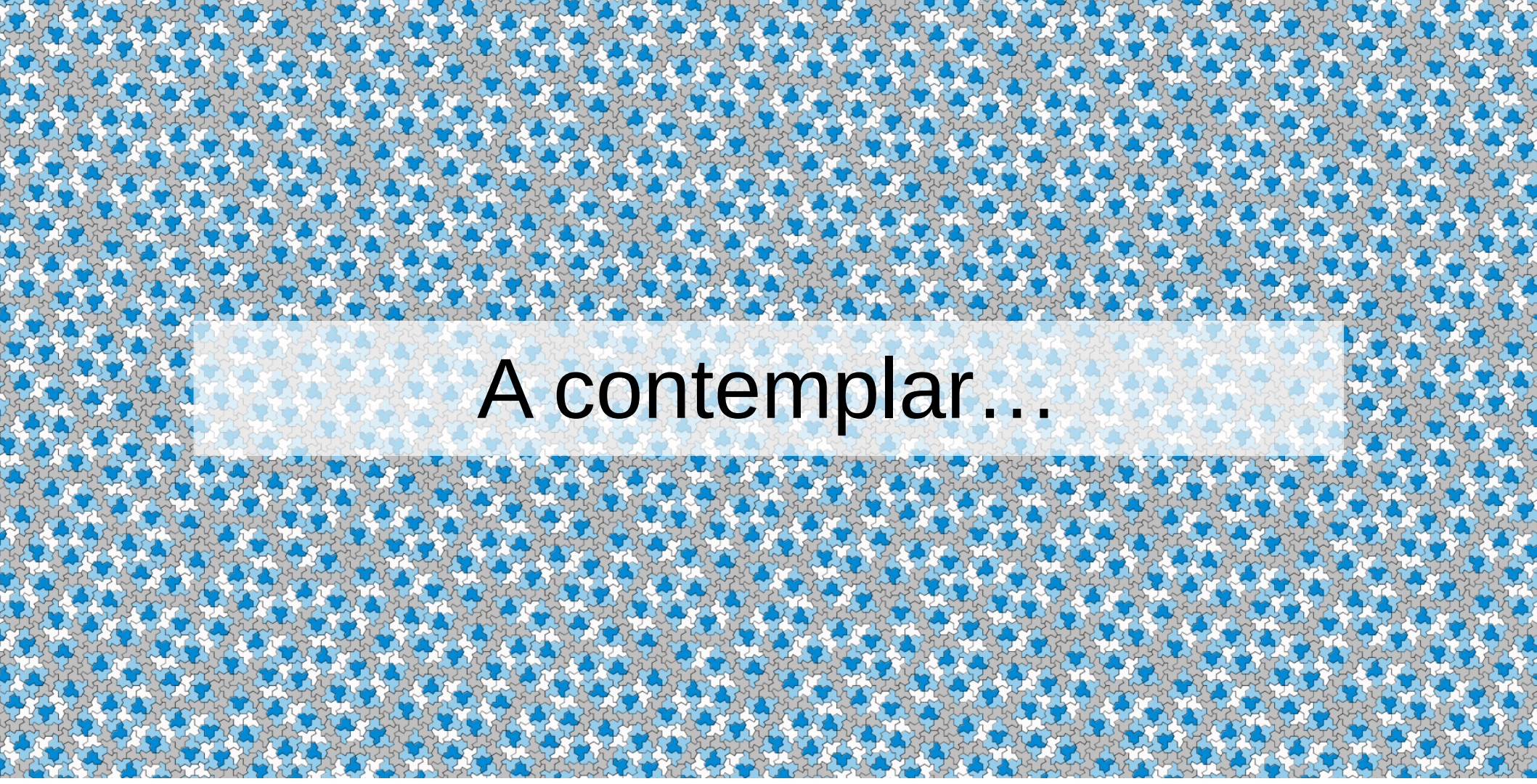
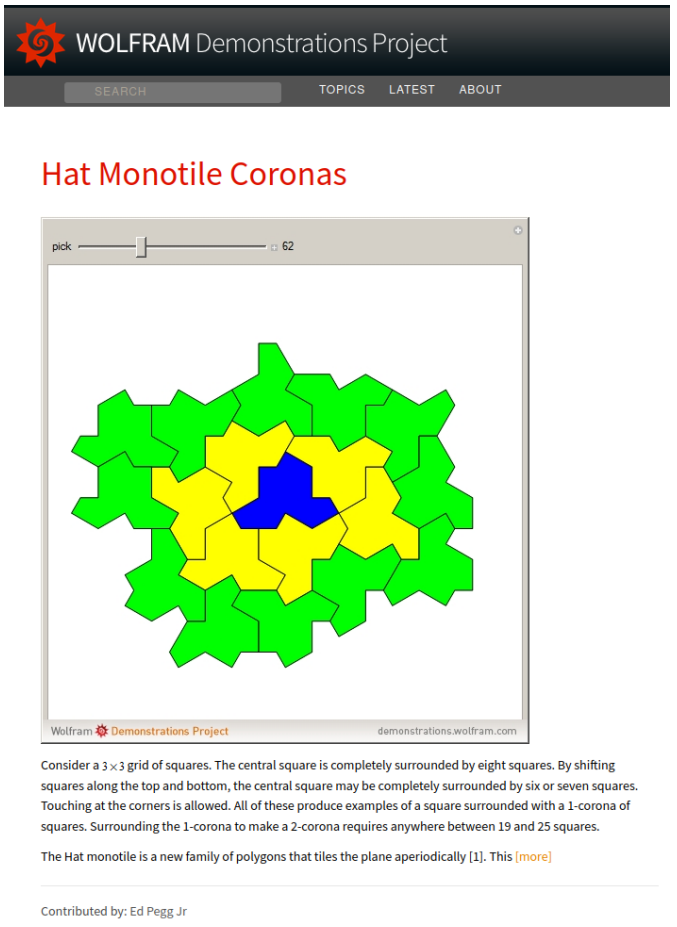


https://cs.uwaterloo.ca/~csk/hat/app.html

...y a jugar!


Robert Fathauer @RobFathauerArt · Mar 21
The new aperiodic monotile discovered by Dave Smith, Joseph Myers, Craig Kaplan, and Chaim Goodman-Strauss, rendered as shirts and hats. The hat tiles are mirrored relative to the shirt tiles.
26
296
1,143
85.6K


WOLFRAM Demonstrations Project
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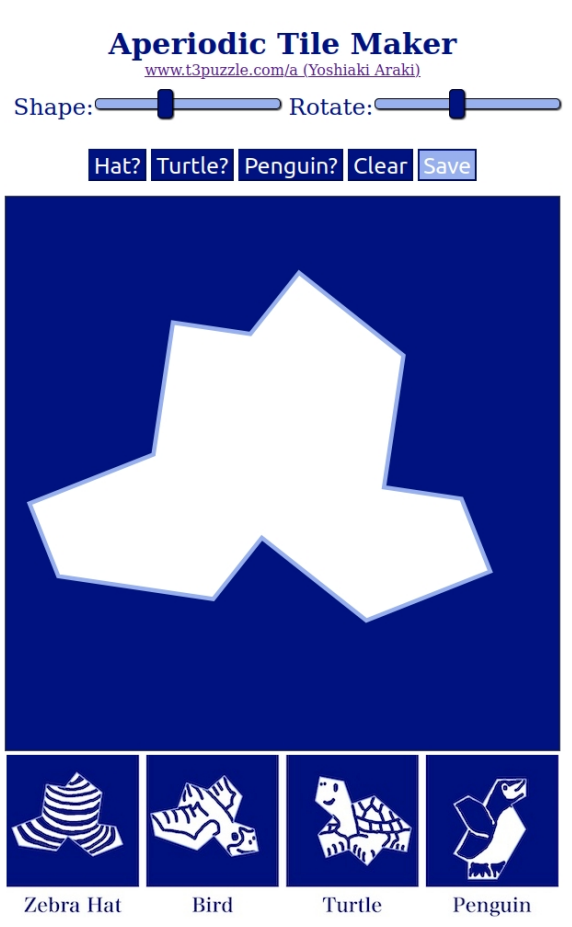
LATEST
ABOUT
Hat Monotile Coronas
pick
62
Wolfram Demonstrations Project
demonstrations.wolfram.com
Consider a 3 × 3 grid of squares. The central square is completely surrounded by eight squares. By shifting squares along the top and bottom, the central square may be completely surrounded by six or seven squares. Touching at the corners is allowed. All of these produce examples of a square surrounded with a 1-corona of squares. Surrounding the 1-corona to make a 2-corona requires anywhere between 19 and 25 squares.
The Hat monotile is a new family of polygons that tiles the plane aperiodically [1]. This [more]
Contributed by: Ed Pegg Jr


Aperiodic Tile Maker
www.t3puzzle.com/a (Yoshiaki Araki)
Shape:
Rotate:
Hat?
Turtle?
Penguin?
Clear
Save
Zebra Hat
Bird
Turtle
Penguin

Sign up to be notified!

MoMath and UKMT to announce a creative artwork competition

In honor of the discovery of the “einstein tile,” the National Museum of Mathematics (MoMath) and the UK Mathematics Trust (UKMT) are jointly organizing a global competition for creative artwork or design. Sign up to be notified when the announcement is released.

Sign up

Aperiodic tiling of turtles across a plane, courtesy of Yoshiaki Araki.

Con ustedes, un...

Gracias!

¿sombrero?

José Ezequiel Soto Sánchez

COLOQUIO DE MATEMÁTICAS